

## **Ball/ Racket Impact and Computer Aided Design of Rackets**

Yoshihiko Kawazoe,  
*Saitama Institute of Technology, 1690 Oaza Fusaiji, Okabe-cho, Osato-gun, Saitama  
369-02, Japan*

### **Abstract**

The coefficient of restitution is closely related to the impact of energy loss. Racket vibration induced by impact seems to be one of the main sources of energy loss. The impact force and contact duration have a strong influence on the racket vibrations. In this paper, the mechanism of impact has been investigated, and a simple impact model has been proposed on the basis of the idea that the contact duration is determined by the natural period of a whole system composing the mass of ball, the nonlinear stiffness of ball and strings, or rubber, and the reduced mass of racket at the impact point on the face. This simple model can explain the experimental results fairly well.

### **1. Introduction**

There are a number of unclarified points regarding impact phenomenon between a ball and a racket as well as regarding optimum design of rackets. It is the purpose of this study to investigate the impact phenomena in racket sports like tennis and table tennis as a first step toward the establishment of racket dynamics and the proposal of a computer aided evaluation system for optimum design of rackets.

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In this paper, the mechanism of impact has been investigated and a simple impact model has been proposed on the basis of the idea that the contact duration, which has a strong influence on the racket vibration, is determined by the natural period of a whole system comprising the mass of the ball, the nonlinear stiffness of ball and strings, or rubber, and the reduced mass of racket at the impact point on the face. This simple model can explain the experimental results fairly well.

### **2. Impact Phenomena in Tennis**

#### **2.1 Coefficient Of Restitution Inherent to the Materials of Ball and Strings**

Measured Coefficient Of Restitution( COR ) *etc* with the racket head clamped is shown in Fig.1(versus the incident velocity). The value of COR is almost independent of ball velocity and string tension. Thus this constant value of COR can be regarded as COR inherent to the materials of ball and strings. The energy loss due to the impact between a ball and strings with racket head firmly clamped is almost independent of impact velocity and tension of strings.

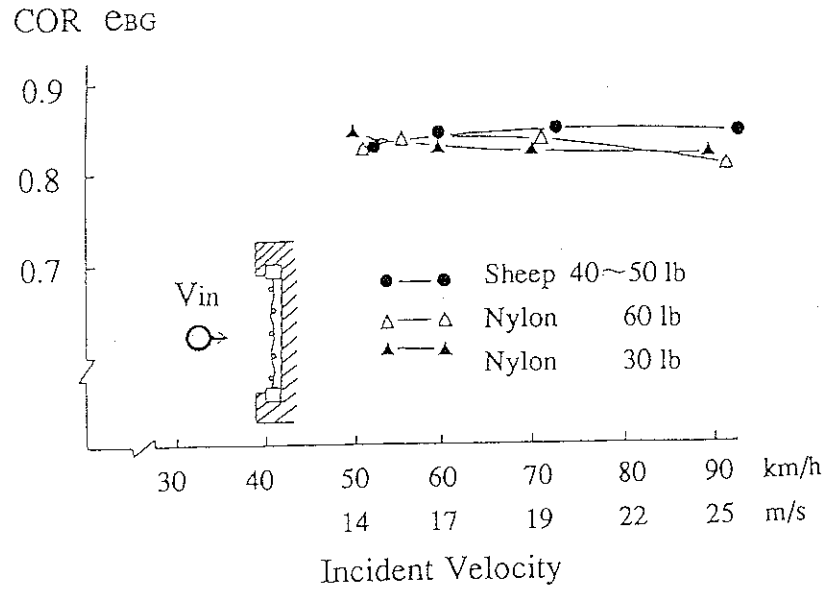


Fig.1 Measured coefficient of restitution between ball and strings with frame fixed.

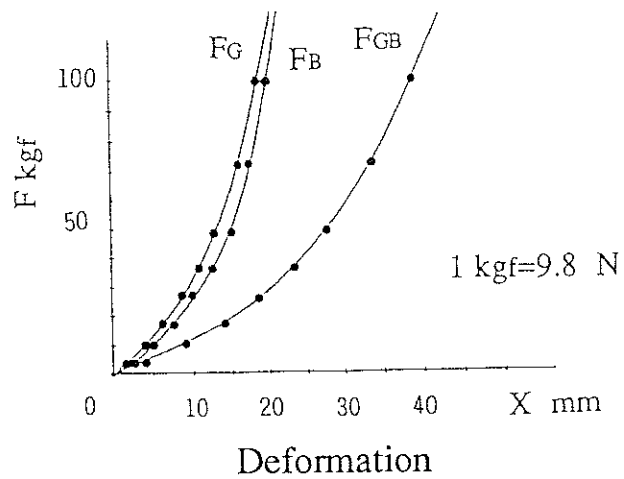


Fig.2(a) Restoring forces v. deformation of ball and strings.

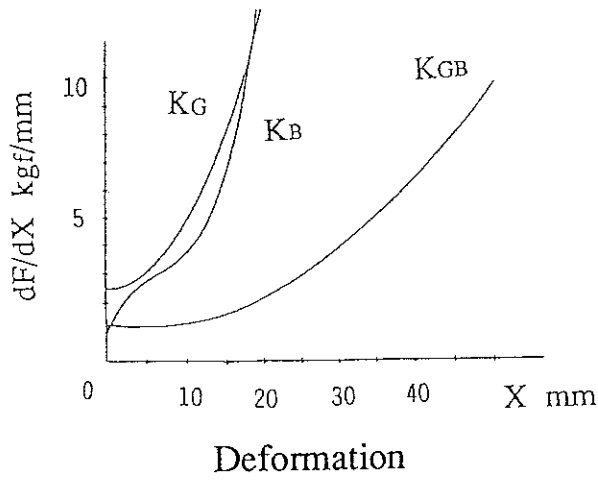


Fig.2(b) Stiffness v. deformation of ball and strings.

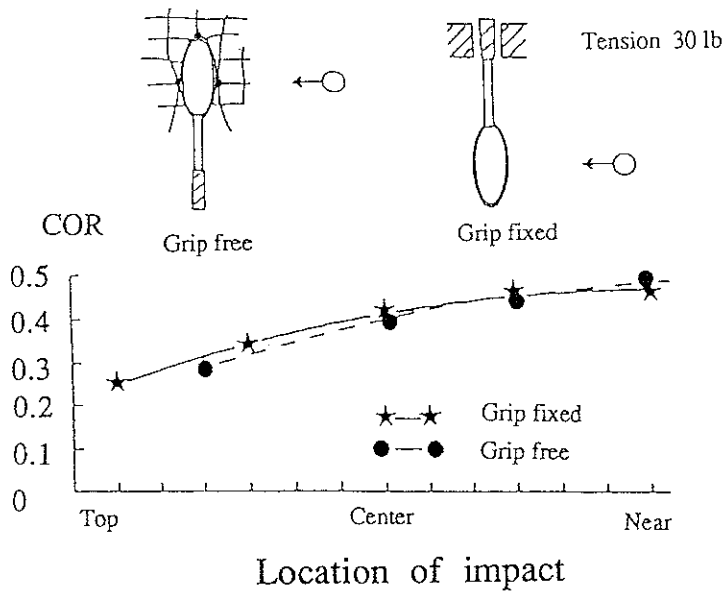


Fig.3 Measured coefficient of restitution at the strings face.

## 2.2 Nonlinear Characteristics of Restoring Force of Ball and Strings

Figure 2(a) shows the curves of restoring force  $F_B$  vs. ball deformation, restoring force  $F_B$  vs. string deformation, and the restoring force  $F_{GB}$  vs. deformation of the compound system of ball and strings, where the curves are determined so as to satisfy a number of experimental data using the least square method assuming that a ball with concentrated mass deforms only at the side in contact with the strings. Figure 2(b) shows the curves of the corresponding stiffness derived by differentiation of the restoring force with respect to deformation.

The springs of both ball and strings become stiffer for larger deformation.

## 2.3 Measured COR when a Ball Strikes the Racket

Figure 3 shows the measured COR when a ball strikes the strings at various places with racket handle firmly clamped or freely supported. It is found that the COR tends to maximize along the longitudinal axis of the racket and peaks close to the throat. The COR curve tends to become lower at the tip and at the near of the racket face.

## 2.4 Measured Contact Duration

Figure 4 shows the measured contact duration which means how long the ball stays on the strings with a normal racket and with a wide-body racket(stiffer). It can be seen that the stiffness of the racket frame does not affect the contact duration much.

# 3. Modelling of Tennis Racket and Prediction of Response due to Impact

## 3.1 Concept of Modelling

With tennis rackets, the mass of ball and racket and the nonlinear stiffness of ball and strings are the main factors which decide the contact time, whereas the stiffness of racket frame and the vibrations of frame and strings do not affect it much. After calculating the impact force and contact time with a model, assuming that a ball collides with a racket frame without vibration through the nonlinear spring, the response of racket can be simulated by applying the impact force to the experimentally identified racket vibration model. The larger the energy loss due to the frame vibration, the lower the coefficient of restitution might be. Also with the table tennis racket, the impact model seems to be almost the same as in tennis if one considers the nonlinear stiffness of rubber instead of strings.

## 3.2 Impact Time and Impact Force against Impact Velocity

The coefficient of restitution  $e_{BG}$  in the collision between a ball and strings with the racket head clamped could be regarded as the instance when the mass of racket frame is infinitely large. In case the vibration of racket frame is neglected, the momentum equation and  $e_{BG}$  give the velocity of a ball following the impact. When a ball collides with a racket standing still before impact, the coefficient of restitution  $e_1$ , defined as the ratio between the relative speeds after and before impact, is easily derived.

Figure 5 shows the calculated COR assuming the racket frame without vibrations, where the reduced mass  $M_r$  is derived in case the racket rotates around the point 70 mm from the grip end. The horizontal axis of Fig.5 means hitting the position on the racket face. It is found that the COR tends to maximize along the longitudinal axis of the racket and peaks close to the throat.

The stiffness  $K_{GB}$  of the ball/strings compound system has a strong nonlinearity and its value changes during impact, depending on the impact velocity. In order to make the

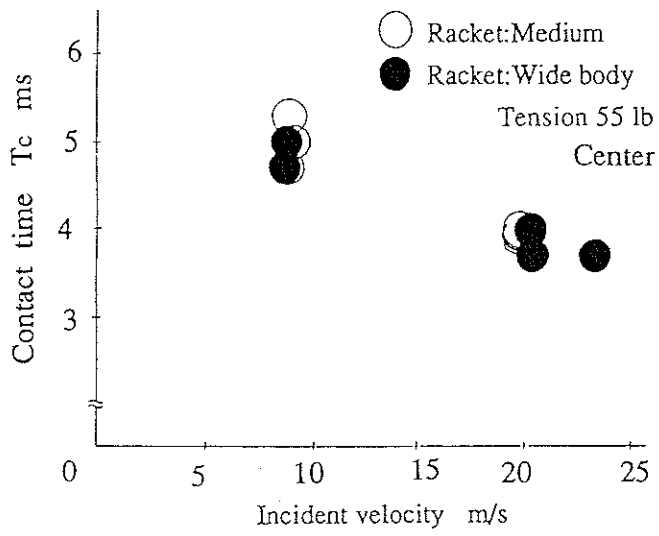


Fig.4 Measured impact duration

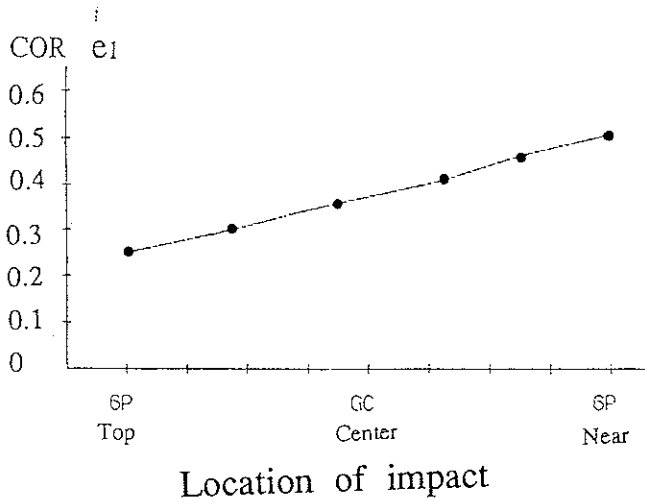


Fig.5 Calculated coefficient of restitution without racket frame vibration (G.C.:Geometrical Center).

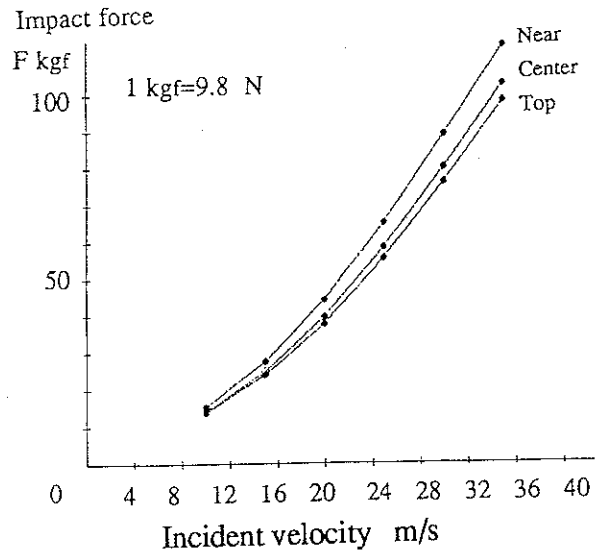


Fig.6(a) Calculated Impact force v. impact velocity

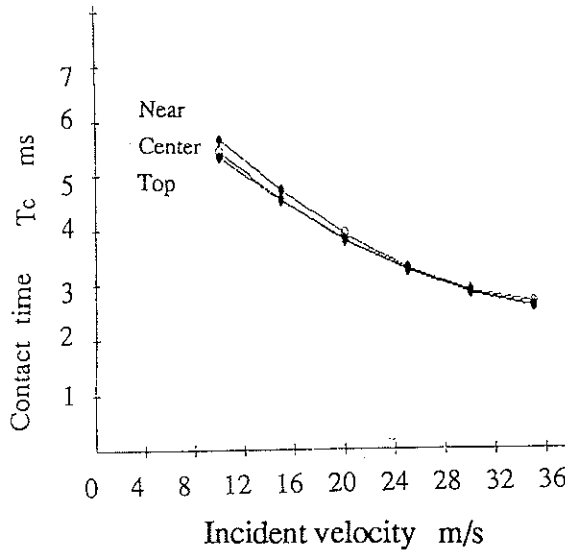


Fig.6(b) Calculated Impact duration v. impact velocity

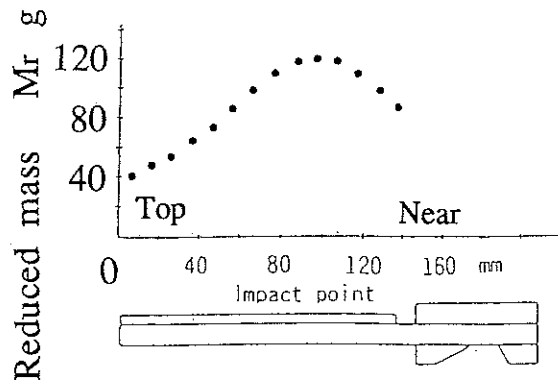


Fig.8 Reduced mass of table tennis racket  $M_r$

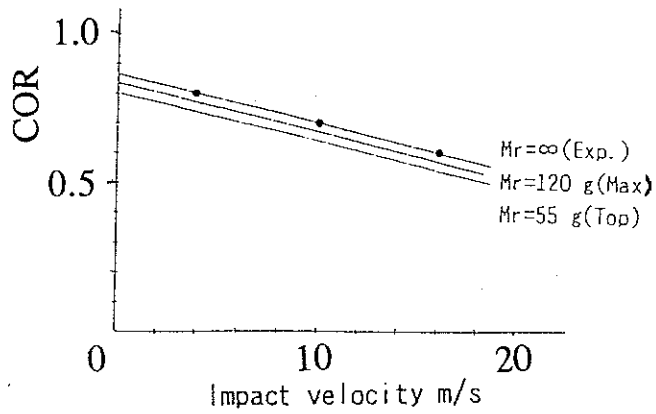


Fig.9 Calculated Coefficient Of Restitution of table tennis racket without racket vibration.

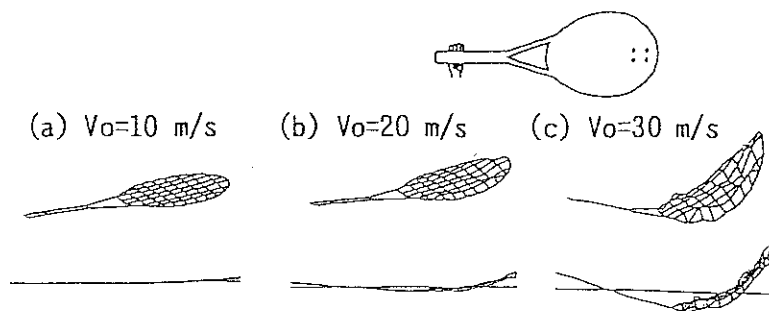


Fig.7 Amplitude Prediction of post impact vibration. (when the ball hits the top,  $V_o$ : Impact velocity)

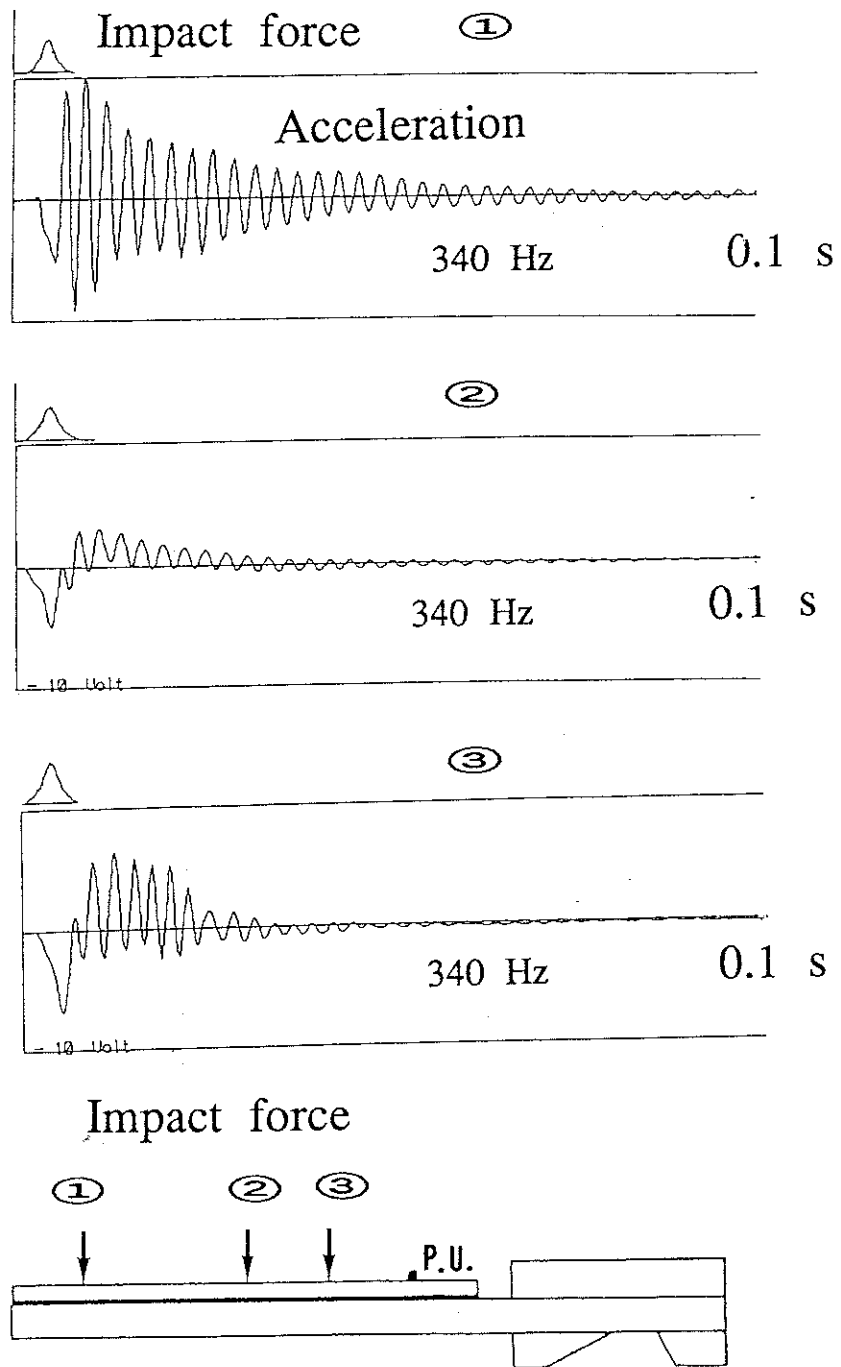


Fig.10 Effect of impact points on the vibrations of racket.



analysis simpler, the mean force  $F_{MEAN}$  can be introduced during impact time  $T_c$ . Thus, if the velocity before impact is given, the relationship between  $F_{MEAN}$  and corresponding  $K_{GB}$  is derived.

On the other hand, the relationship between  $F_{GB}$  and  $K_{GB}$  can be derived from Fig.2(a) and Fig.2(b). From these two relations, the parameters  $K_{GB}$ ,  $F_{MEAN}$ ,  $T_c$  and deformation of ball and strings can be determined against the velocity before impact.

The calculated impact force  $F_{MEAN}$  and contact duration  $T_c$  against the velocity before impact are shown in Fig.6(a) and Fig.6(b) respectively.

### 3.3 Approximation of Impact Force–Time Curve

Since the force–time curve of impact has an influence on the magnitude of racket frame vibration, it is approximated as a half–sine pulse, which is similar in shape to the actual impact force.

### 3.4 Prediction of Racket Vibration Against the Impact Velocity

When the hitting position and the velocity before impact are given, the vibration of the racket can be simulated by applying the impact force to the experimentally identified racket model(1). Figure 7 shows the amplitude of racket displacement when a ball strikes the top of the racket face at a speed of 10, 20, and 30 m/s.

### 3.5 Estimation of Racket Performance

Racket performance with respect to the coefficient of restitution might be estimated from the energy loss due to post impact vibrations. The energy loss is very small when a ball hits the center, whereas it is large when a ball hits the top or the near of the racket face.

The measured COR (Fig.3), when the center is hit is very close to the calculated COR without consideration of frame vibrations(Fig.5), and the measured one, is lower than the calculated one when the off–center is struck. This means the larger the energy loss due to frame vibration the lower the COR is. The effect of impact velocity, mass balance, tension of strings, and stiffness of frame on the impact behavior could be estimated using this simple model.

## 4. Impact Phenomena in Table Tennis

### 4.1 Reduced Mass of Table Tennis Racket at the Impact Point

Figure 8 shows the reduced mass of table tennis racket when the racket rotates around the center of gravity at impact. Although the mass of a racket is quite large compared with the mass of a ball, the reduced mass of a table tennis racket depends on hitting locations.

### 4.2 Calculated COR of Table Tennis Racket Without Racket Vibrations

Figure 9 shows the calculated COR of a table tennis racket when struck at the top and near (close to the center of gravity) of the racket without vibrations, compared to the experimental results (2) where a ball hits the rubber stuck to the lead.

### 4.3 The Effect of Hitting Point on the Vibrations of Table Tennis Racket

Figure 10 shows the accelerations of rubber when an impulse hammer hits the top, the center, and the near of the racket face. The amplitude of vibration is large when the

top is struck, whereas rather small when the center is struck. This behavior is similar to that of a tennis racket. Since the contact time is much shorter in actual impact, vibration of racket will be much larger, accordingly the COR might be lowered due to the energy loss induced by impact.

## **5. Conclusions**

The mechanism of impact has been investigated, and a simple impact model in tennis and table tennis has been proposed on the basis of the idea that the contact duration, which has a strong influence on the racket vibration, is determined by the natural period of a whole system composing the mass of ball, the stiffness of ball and strings, or rubber, and the reduced mass of racket at the impact point on the face.

The experimental results on the impact in tennis can be explained fairly well with this simple model. Additionally, a few experiments and remarks have been given on the impact in table tennis.

## **Acknowledgments**

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