

## **Perfecting of a Ball Bounce and Trajectories Simulation Software :In order to Predict the Consequences of Changing Table Tennis Rules**

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### **Abstract**

The purpose of this paper is to present the software that we are perfecting to simulate the game of table tennis. First we described the ball bounce and trajectories models used in a first software functioning at present . We gave a few examples of results achieved due to this software concerning ball effects. Then we proposed to integrate the racket-ball impact model into this software as well as elements of the stroke construction model.

Key words :Impact, model, trajectory, software, stroke, system

### **1- Modelling of ball trajectories**

The model used is the model designed by R.Seydel in his thesis (Seydel, R. 1990) which is described in the previous journal (Seydel, R. 1992) Three forces are exerted on the ball :  $\vec{D}$  Drag force,  $\vec{L}$  Lift force,  $\vec{G}$  Gravity (Fig. 1).

This model enables the values of the forces at any time to be determined due to the experimental study of aerodynamical coefficients  $C_{DA}$  and  $C_{LA}$  to draw the ball trajectories. Otherwise the model makes it possible to understand qualitatively the effects on the ball. These effects derive from  $\vec{L}$  force which is perpendicular to the velocity. This model gives the direction of the ball which is the same as the direction of vector  $\vec{n} = \vec{\omega} \wedge \vec{V}$ . From this definition we can infer "basic" effects.

The direction of the force  $\vec{L}$  responsible for the effect is defined by the cross product of the spin vector  $\vec{\omega}$  and the velocity  $\vec{V}$ . To find it we only need to move the vector  $\vec{\omega}$  on to the vector  $\vec{V}$ . This rotation induces a moving direction in a direct frame.

This moving direction is in the direction of the vector  $\vec{L}$ . We can thus infer the effects on the trajectory. For instance :

- if  $\vec{\omega}$  is orientated upwards the deviation will be on the left and vice versa
- if  $\vec{\omega}$  is orientated downwards the deviation will be on the right.

• if the rotation axis is horizontal (parallel to the net) the deviation will be orientated downwards (Topspin if  $\vec{\omega}$  is positive) or upwards (backspin if  $\vec{\omega}$  negative) (Fig. 2).

1.1-Definition of inputs for the calculation of the ball trajectory (Fig. 3)

1.2-Definition of velocity inputs (Fig. 4)

$\vec{V}$  : Ball velocity vector at the starting point of the trajectory.

$|V|$  is the speed value

$A_V$  vertical angle

$A_L$  lateral angle

Then the software calculates the components of the velocity :

$$V_x = V \cos(A_V) \sin(A_L)$$

$$V_y = V \cos(A_V) \cos(A_L)$$

$$V_z = V \sin(A_V)$$

1.3- Definition of rotation inputs

$\vec{\omega}$  : Ball rotation vector at the starting point of the trajectory (Fig. 5).

$|\omega|$ : rotation modulus

$\omega = 2\pi \cdot N$  (  $N$  is expressed in revolutions per second )

$q$ = frontal plane angle

$f$ = angle in relation to this frontal plane

Then the software calculates the components of the rotation.

$$\omega_x = \omega \sin(f)$$

$$\omega_y = -\omega \cos(f) \sin(q)$$

$$\omega_z = \omega \cos(f) \cos(q)$$

We find the basic effects again when two of these components are zero.

1.4- Parameters influencing the trajectory.

The model permits the calculation of the sequence of the positions of the ball successively regarding a constant set of inputs. Therefore it is necessary to specify the values of the model parameters i.e. the values of the aerodynamical coefficients, the mass, the radius of the ball and the volumetric mass of air. We are thus able to measure the variation effects of these parameters and we are able to compare the induced trajectories for different values of the parameters for the same inputs.

## 2- Ball bounce modelling

The bounce is a short instant ( a few milliseconds) that induces a change in velocity. The bounce can be "modeled" taking account of the global changes of the inputs. The model of the bounce is given by the impact of a stiff sphere on a plane. The impact is characterized by the impulse, global loss of energy and forces of interaction (Fig. 6).

We have defined the coefficient of restitution as the ratio of relative velocities before and after the impact in the perpendicular direction to the contact plane :

$$e = -\frac{V_y}{V_y}$$

All the forces of interaction during the impact are reduced to a torque and a resultant force ( $\vec{R}$ ,  $\vec{C}$ ) (Fig. 7).

The contact duration of the ball on the table is written  $t$ .

So we can write the impulse force :  $\vec{P} = \int_0^t \vec{R} dt$

The general laws of dynamics give under their integrated form the following equations :

Newton's second law :

$$\vec{P} = m \cdot \Delta \vec{V}$$

Law of conservation of angular momentum

$$\vec{G}O \wedge \vec{P} + \vec{\Gamma} = I \cdot \Delta \vec{\omega}$$

$$(Fig. 7) \quad \text{with} \quad \vec{\Gamma} = \int_0^t \vec{C} dt \quad \text{and } I \text{ inertial moment}$$

## 2.1-Results

The behavior of the ball during the impact, that is its rolling without sliding (rolling) or its rolling and sliding (sliding) depends on the velocity of the point of contact of the ball in relation to the table. The relative velocity can be characterized quantitatively by the spin parameter (SP). This parameter is defined by the ratio of the velocity of a peripheric point of the ball to the velocity of the center of gravity of the ball (Fig. 8) :

$$SP = -\frac{r\omega_y}{V_x} = -\frac{\text{velocity of a peripheric point of the ball (on } O_x \text{ axis)}}{\text{velocity of center of gravity of the ball (on } O_x \text{ axis)}}$$

## 2.2- Instance of use of the spin parameter

In the case of a topspin ball (as above vector  $\omega_y$ ), the ball rotation induces a peripheric velocity of the points of the ball located at the contact point  $I$  as opposed to the translation velocity of the ball. The relative velocity of this point of the ball in relation to the table is lower than the velocity of the center of gravity of the ball (for a backspin it is the opposite phenomenon).

If  $r\omega_y = -V_x$  the point of the ball in contact with the table has a relative velocity equal to zero.

- For topspin the spin parameter is positive.
- When there is no rotation its value is equal to zero
- For backspin the spin parameter is negative.

## 2.2- Instances of values of the spin parameters for different strokes (Table 1)

ball radius :  $r = 19 \text{ mm}$ 

$$SP = 2\pi r \frac{N}{V}$$

	V (ms <sup>-1</sup> )	N (*)	SP
Chop 3	6	-130	-2.6
Chop 2	10	-130	-1.6
Push 2	8	-60	-0.6
Chop 1	6	-20	-0.4
Push 1	4	-20	-0.4
Smahs 2	18	0	0
Block 2	18	40	0.13
Smash 1	30	60	0.24
Block 1	6	20	0.4
Topspin 3	20	160	0.955
Topspin 2	12	110	1.1
Topspin 1	12	160	1.6

\* N is expressed in revolutions per second

The behavior of the ball during the impact is given by boundary values of the spin parameter (Fig. 9).  $q$  is the velocity angle with the Ox axis before the impact (Fig. 10)

$$\text{Boundary values of } SP = 1 \pm \frac{5f(1+e)}{2} \text{TAN}(\theta) \text{ (Fig. 11).}$$

When measurements have been carried out on tables and balls authorized by the ITTF, we have found  $f = 0.2$  and  $e = 0.8$  and therefore :  $SP = 1 \pm 0.9 \text{TAN}(q)$ .

## 2.3- Synthetic results giving the velocity, rotation and the sliding velocity values after the bounce (Fig. 11).

## 3- Simulation results

## 3-1- Relations between vector velocity and vector rotation (ORFEUIL, F., 1992)

The research deals with the relation between the trajectory and the velocity of the ball. The racket-ball impact is located one meter behind the table baseline and 16 cm under the surface of the plane of the table.

Simulation deals with the determination of two boundary vertical angles (for a defined velocity) of the velocity vector ; the first angle produces a ball with a bounce on the table base line, the second angle produces a skimming ball over the net. We make the same simulation with balls moving on the diagonal of the table. See Fig. 12 the trajectory drawn by the software for the determination of one boundary angle.

Results are summed up in Fig. 13. On this figure we can read boundary angles for a given velocity producing a good ball and vice versa. For instance for a velocity angle of 45° the player's ball can move from 21 Km/h to 26 Km/h.

The more interesting result concerns the rotation effect (Fig. 14). The faster the rotation of the ball, the faster the velocity for the same angles. Thus the player can increase the velocity of his ball by 50%. The player can give an initial velocity of more than 100 km/h in the same initial conditions (1m behind the table baseline and 16 cm under the surface plane of the table).

## 3.2- Definition of basic effects, ball rotation and theoretical form of the trajectories (Fig. -15-16-17-18).

## 3.3- Specific case of bounce producing a deviation after the bounce without initial rotation on Ox (Fig. 19).

At the start the ball has the following rotation : left side effect and topspin.

Because of the curve of the trajectory the plane of the velocity after bounce is located in a frame that has revolved around Oz. The rotation axis is unchanged, but now, in the new frame of the velocity, the rotation vector comprises three components ( $\omega_z$ ,  $\omega'_x$ ,  $\omega'_y$ ). In this frame of reference,  $\omega'_x$  will produce in our example a right deviation after bounce (right jump).

## 4- In order to integrate other models to simulate changes in the game linked with changes in the rules.

A "systemic" point of view on table tennis.

We call the system comprising of all the people involved in the performance of a point (the players and the referee) and the table and the ball in an environment from the serve to a winning point the "point system". These different elements of the system exchange flows between each other that define and organize the dynamic functioning of this system.

These flows indexed in the flow diagram (Fig. 20) modify the state of the subsystems.

In specific cases models of the changes in these subsystems can be designed. We have then a model of functioning of a subsystem in its environment composed by the whole model (Fig. 21).

For instance :

- Model of the ball trajectory linked with the interactions with air and earth (See 1-)
- Model of bouncing of the ball linked with the interaction of the ball with the table (See 2-)

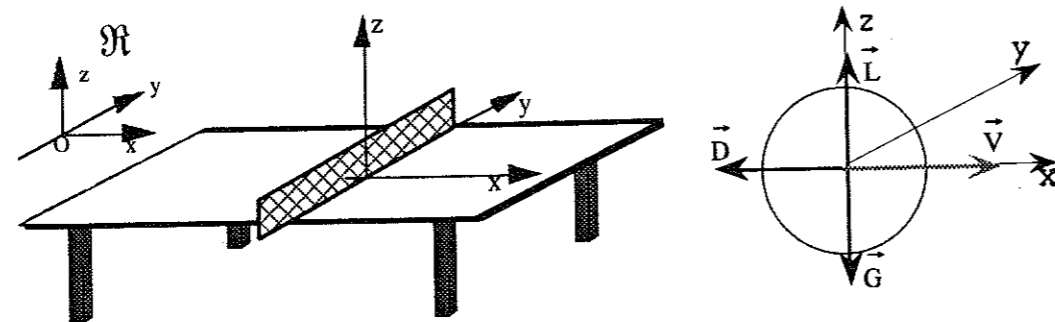


Figure 1 Frame of reference and forces exerted on the ball

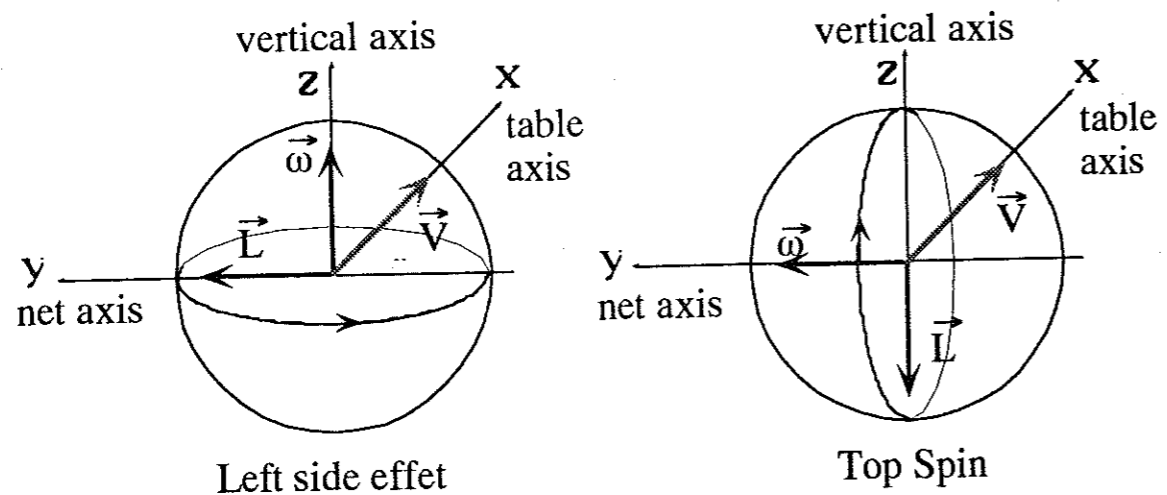


Figure 2 Two basic effects : Left side effect and Top Spin

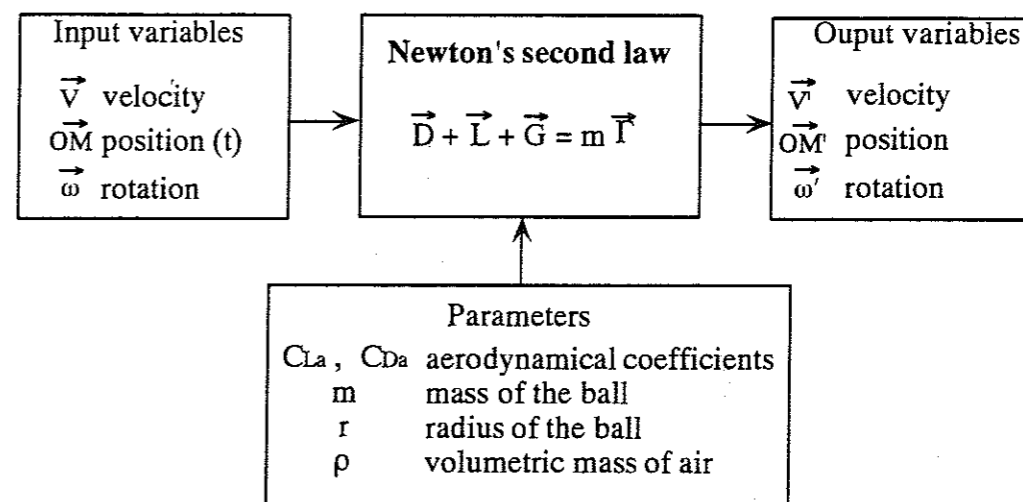


Figure 3 The Trajectory model with variables and parameters

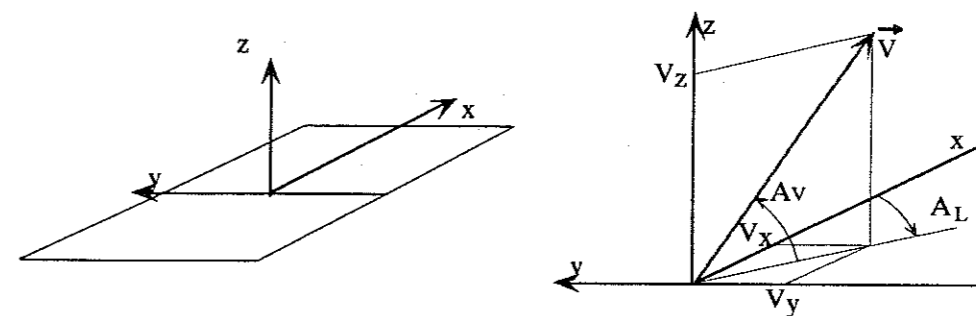


Figure 4 Frame reference and definition of velocity vector components

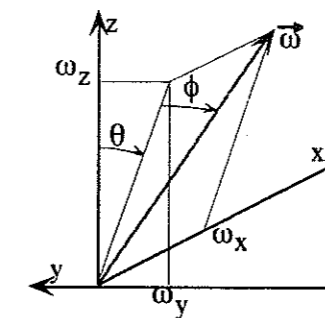


Figure 5 Definition of rotation vector components

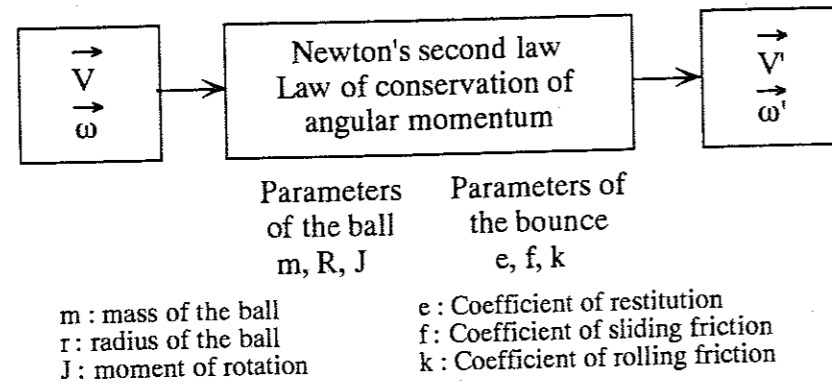


Figure 6 Model of the bounce

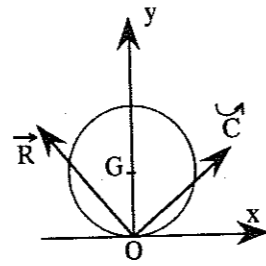


Figure 7 Forces of interaction

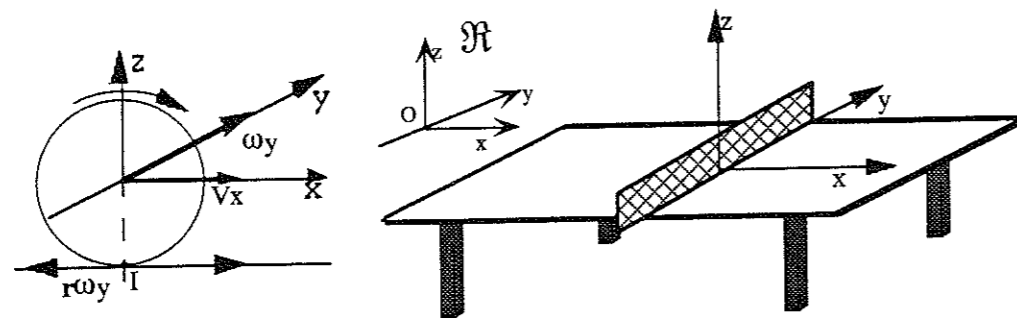


Figure 8 Frame of reference and Spin parameters

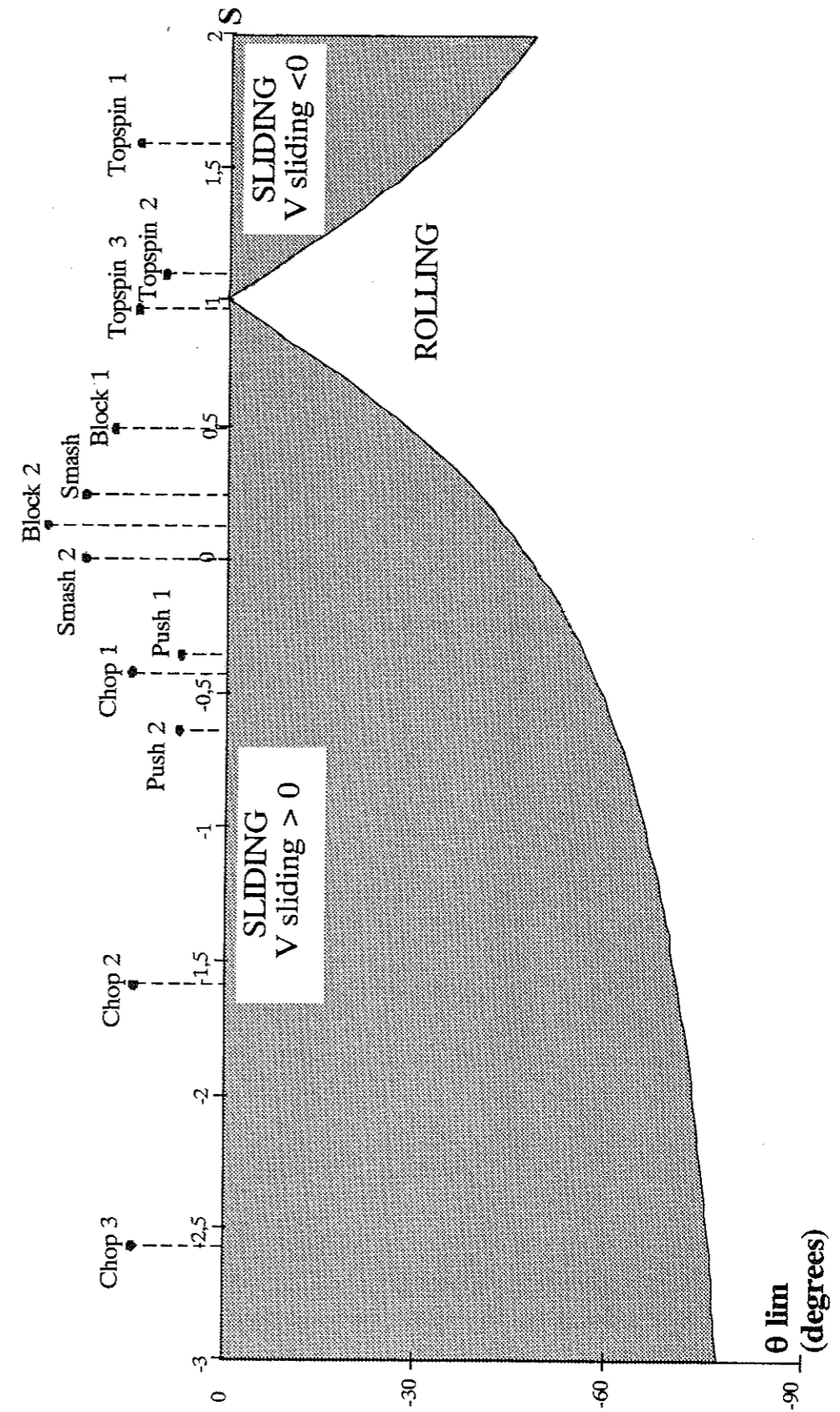


Figure 9 Curve giving the value of the boundary angle in relation to the spin parameter

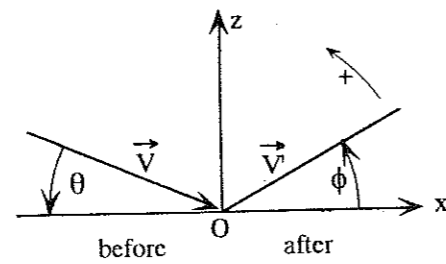


Figure 10 Definition of the impact angles

Sliding velocity >0	Rolling	Sliding velocity <0
Limit value of SP between sliding and rolling		
-3	$1 + \frac{5f}{2}(1+e)\text{TAN}(\theta)$	$1 + \frac{5f}{2}(1+e)\text{TAN}(\theta)$
SP		
$V'_x = V_x + f(1+e)V_z$	$V'_x = \frac{3V_x + 2r\omega_y}{5}$	$V'_x = V_x - f(1+e)V_z$
$\omega'_y = \omega_y - \frac{3f}{2r}(1+e)V_z$	$\omega'_y = \frac{V'_x}{r}$	$\omega'_y = \omega_y + \frac{3f}{2r}(1+e)V_z$
$V_{\text{sliding on Ox}} = (1-B)V_x - r\omega_y$	$V_{\text{sliding on Ox}} = 0$	$V_{\text{sliding on Ox}} = (1+B)V_x - r\omega_y$
$B = -\frac{3f}{2}(1+e)\text{tg}\theta$		

Figure 11 Synthetic results giving the velocity, rotation and the sliding velocity values after the bounce

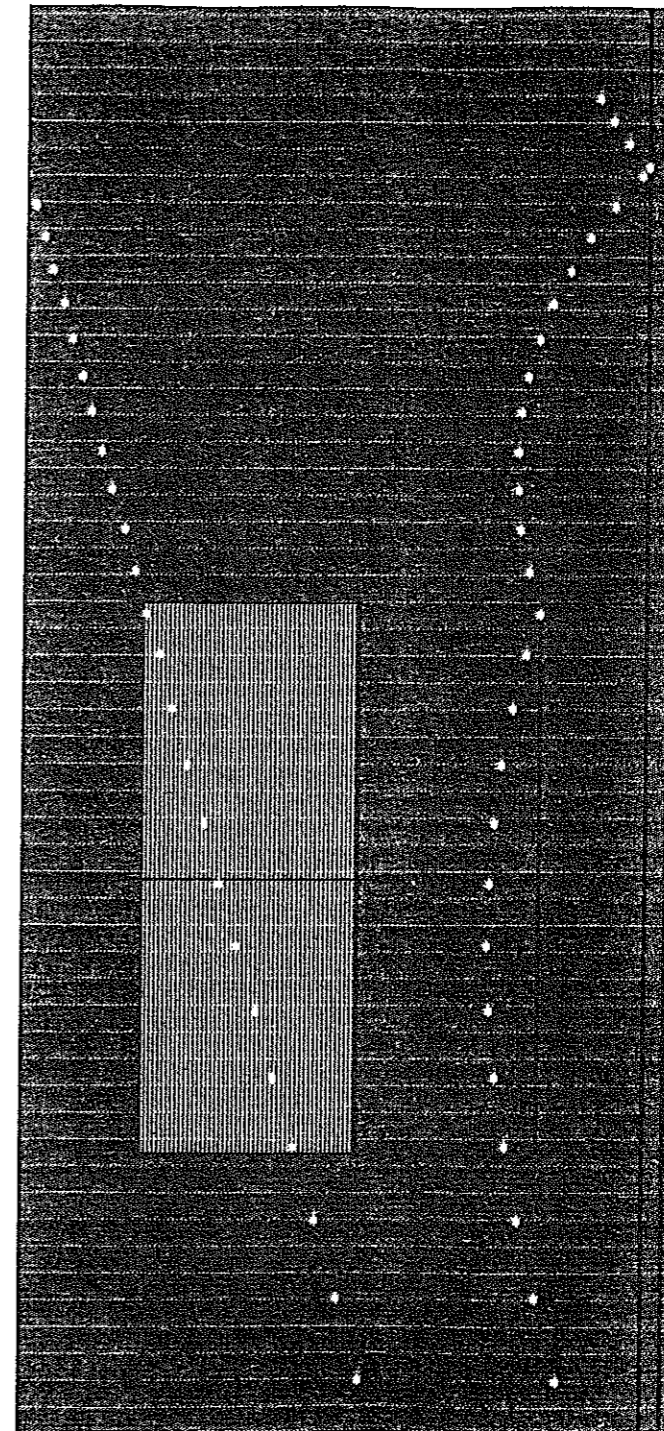


Figure 12 Trajectory drawn by the software of a ball producing a bounce on table base line

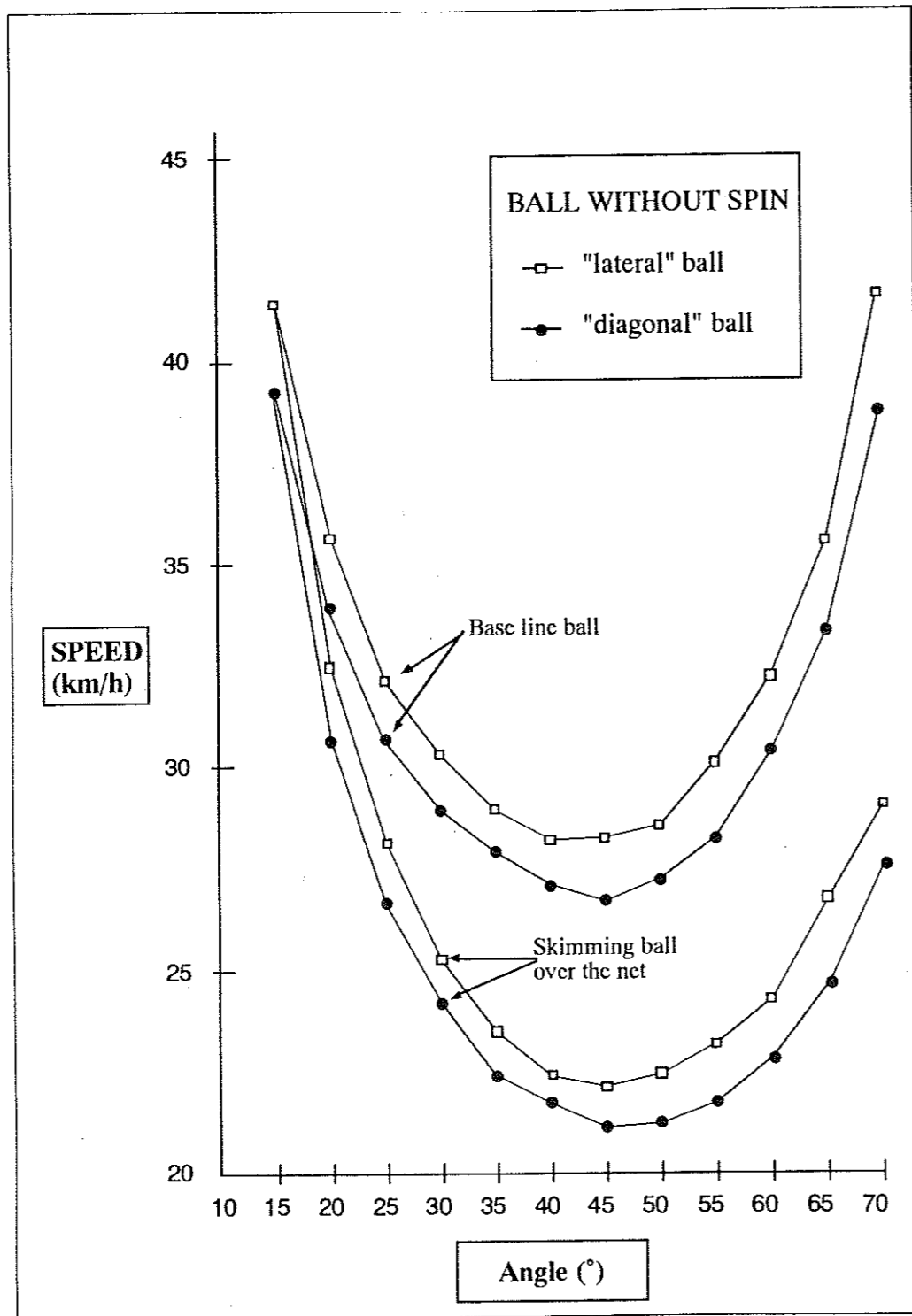


Figure 13 Boundary angles in relation to speed (ball without rotation)

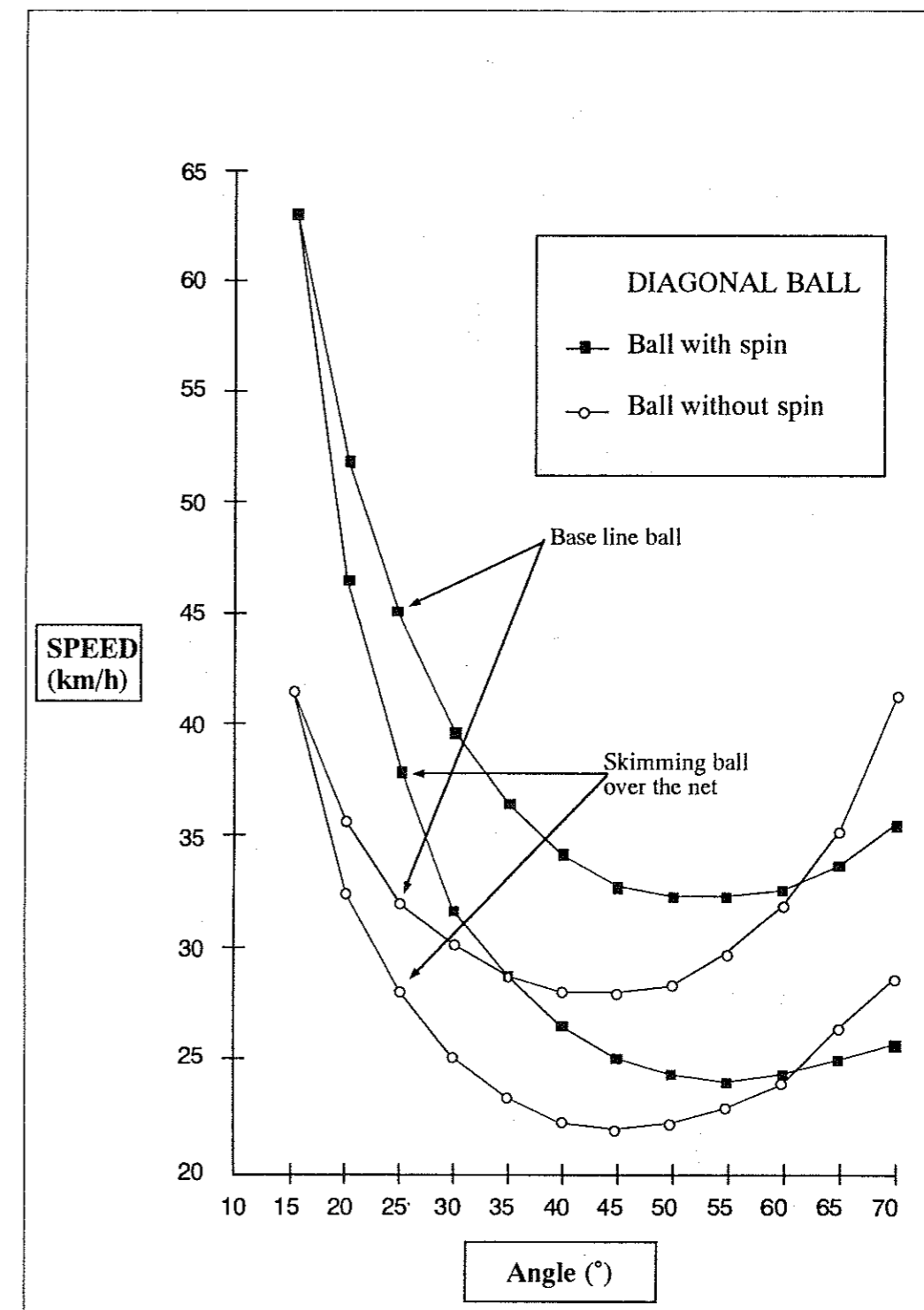


Figure 14 Boundary angles in relation to speed (diagonal ball)

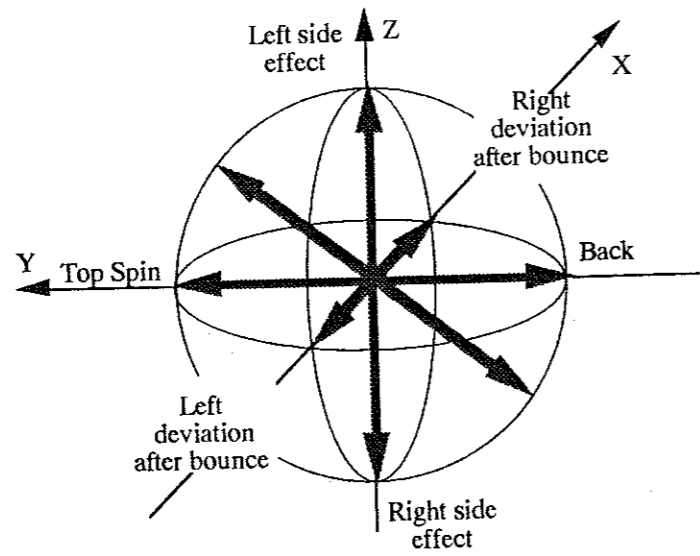


Figure 15 Basic effects definition

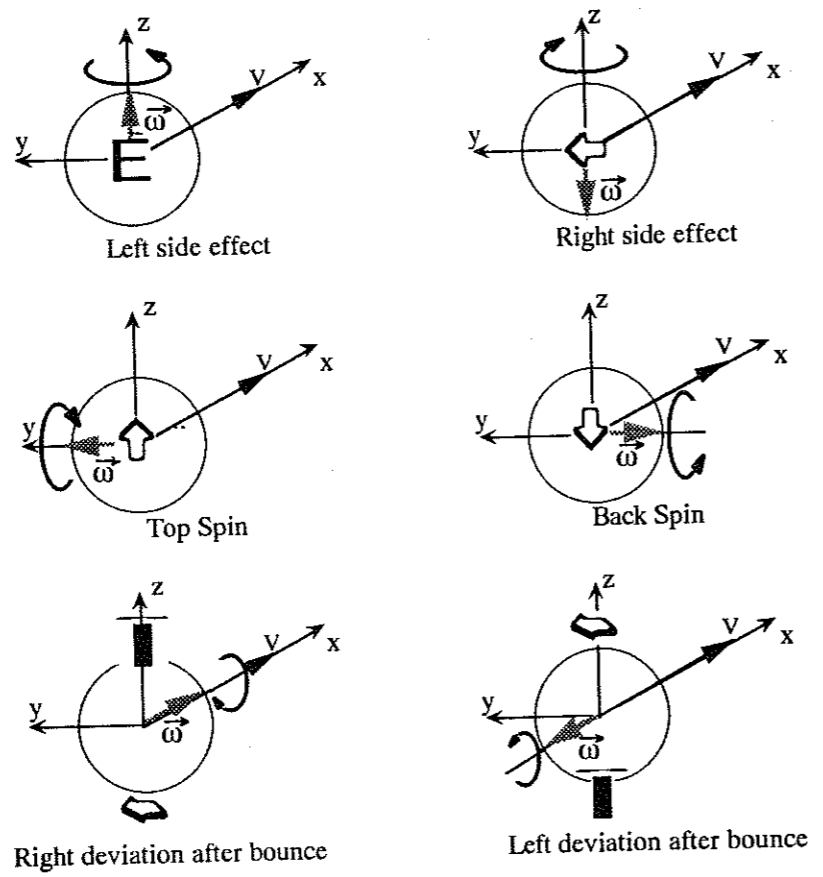


Figure 16 Ball rotation

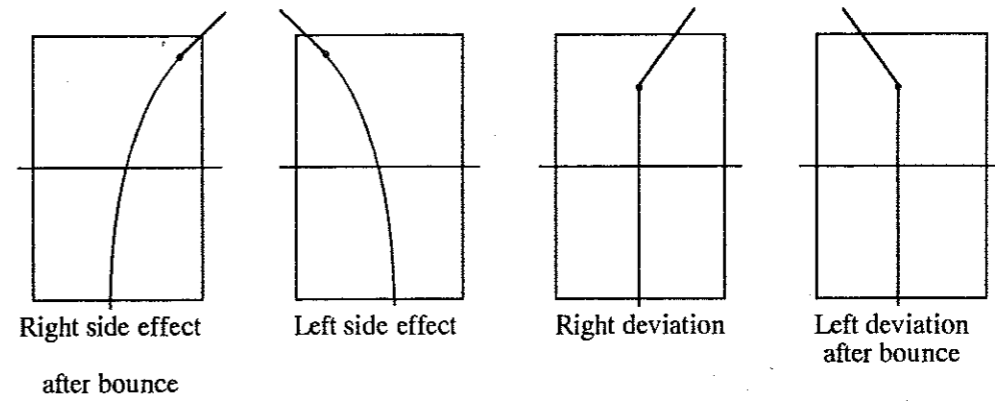


Figure 17 Theoretical form of trajectories with effect

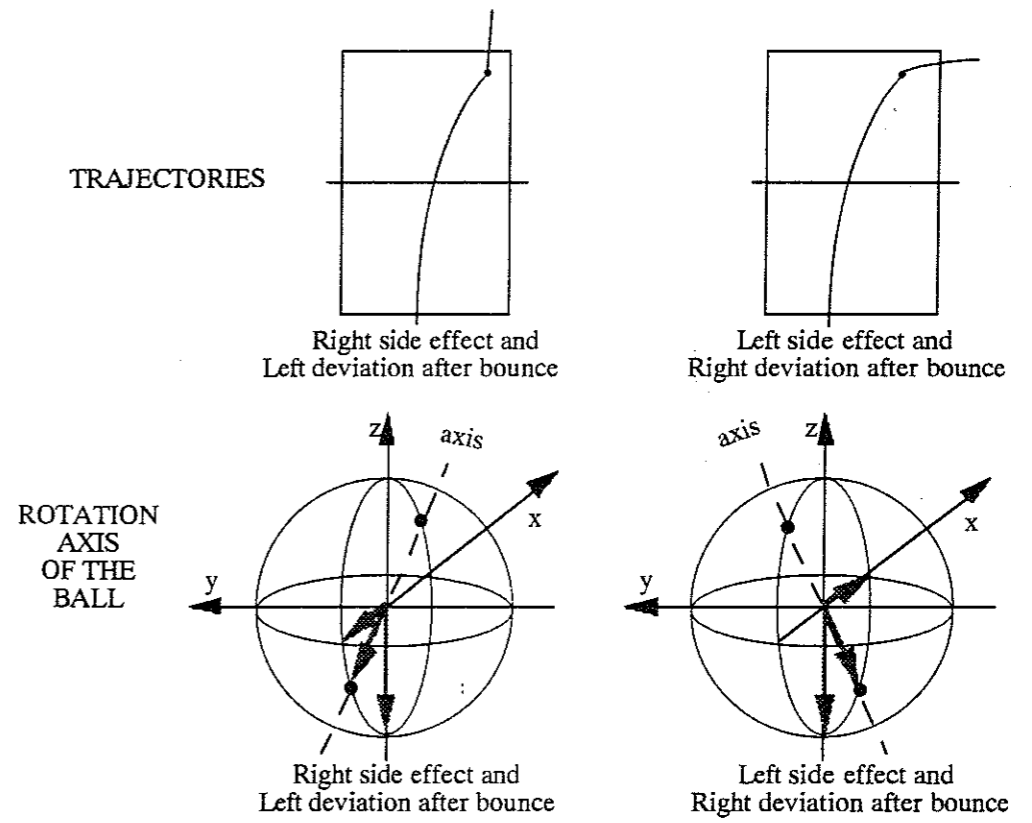


Figure 18 Trajectories with mixed effects



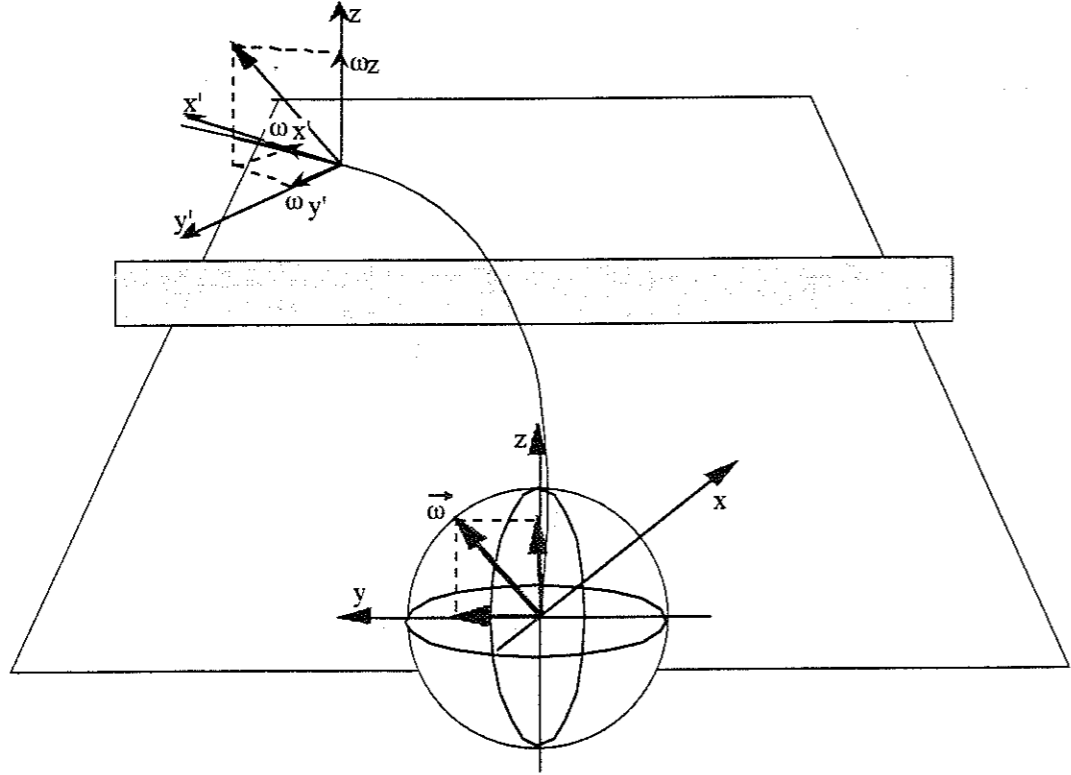


Figure 19 Specific case of bounce producing a deviation after the bounce without initial rotation on Ox

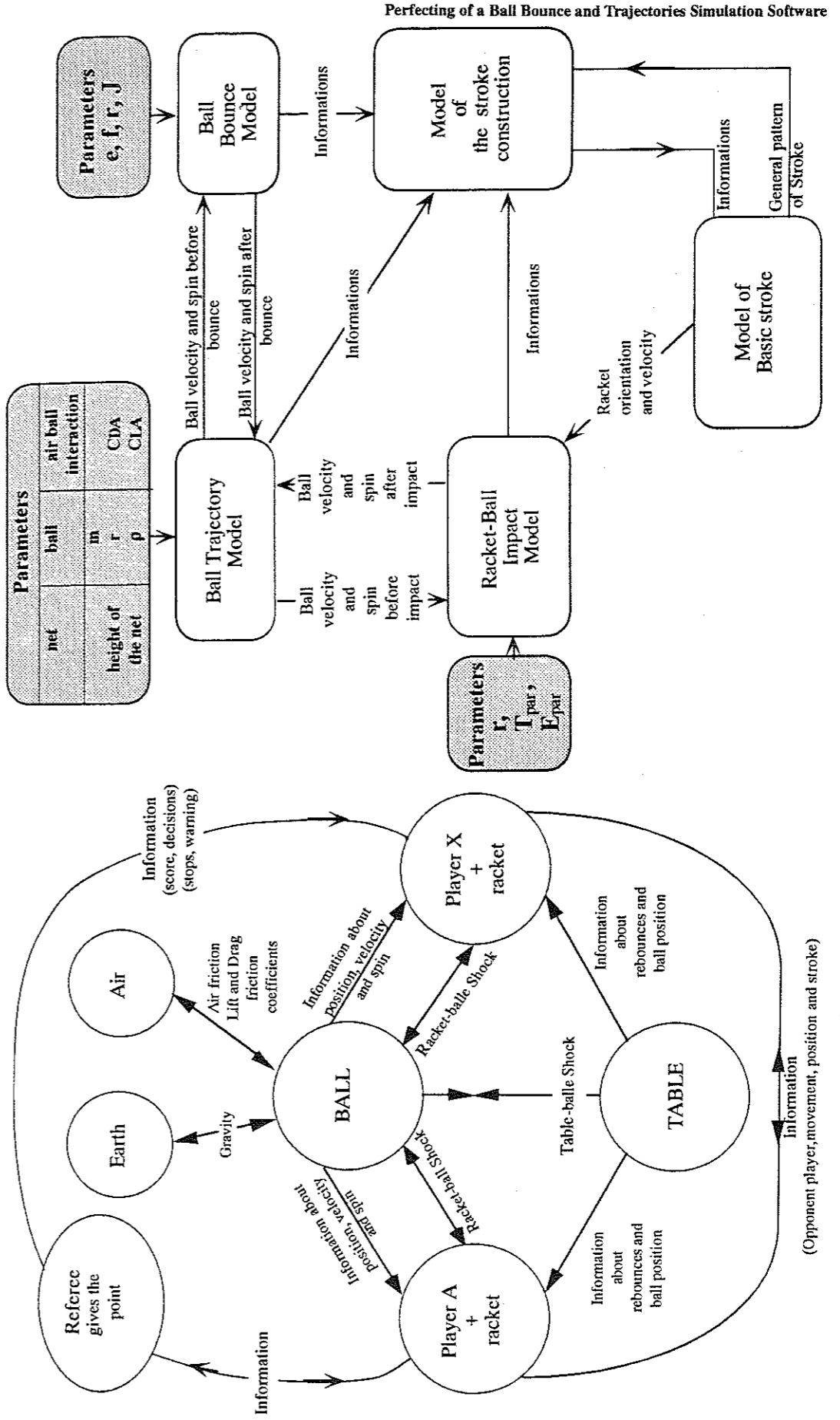


Figure 20 Flow diagram of the "POINT SYSTEM"

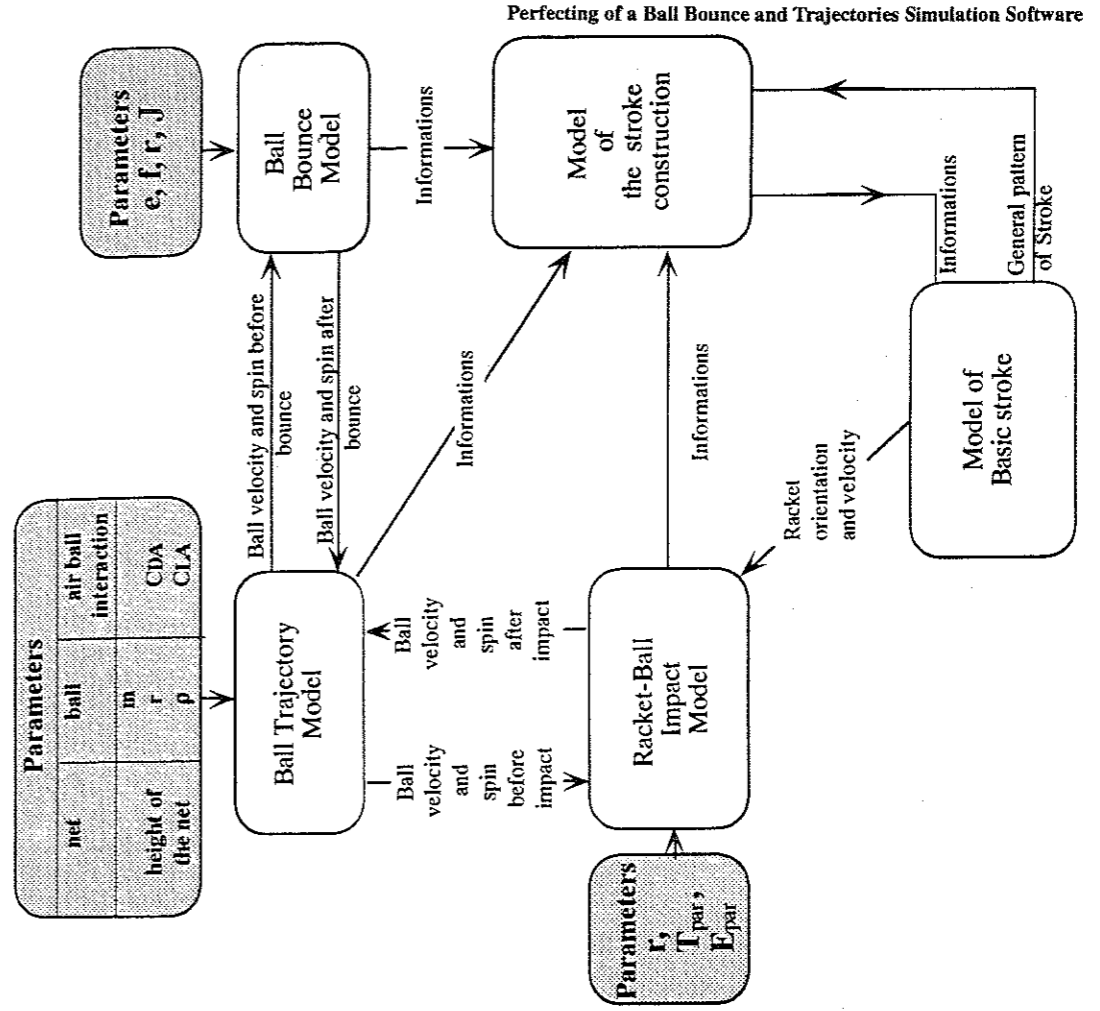


Figure 21 Model of interactions between the different sub-system model to highlight the parameters which can be modified with our model to simulate changes in the game.

- Model of the racket ball impact linked with the interaction of the player and the ball through the racket (Cf in this journal the paper of Tiefenbacher -Durey)
- Model of the stroke construction (Cf in this journal the paper of Ramanantsoa-Durey)

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