Basic theory and experiment for the simulation of ball trajectory

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Abstract

Several interesting problems on the dynamics of the motion of a ball flying in the air and the mechanism of its collision with a surface are investigated by theoretical consideration, production of computer graphic animation and experimental simulation. These methods deal with the complex motion resulting from dynamic interaction between the ball motion and fluid flow surrounding the ball. From this scientific research we could also discuss the human skill or physical ability needed for playing table tennis. In the present paper, recent results of the research for analyzing the ball motion are reported schematically.

1. Introduction

Since a table tennis ball behaves in a very complicated way, as we know well, there are many degrees of freedom and great opportunity to use this complexity in the techniques of play. The top players have very high levels of skill in serving and returning the ball even though their physical power or foot work may be deficient. However, the complexity of the ball motion still needs to be completely explained as scientific problems. It stems from the dynamic interaction between the motion of the ball and that of air flow surrounding it. The mechanisms of ball collisions on the table surface and racket surface are also a very important phenomena governing the ball trajectory. These processes are made clear and analyzed in detail by the application of established knowledge and new methods in the field of fluid mechanics, for example a PIV (Particle Imaging Velocimetry) method proposed by the authors[1] and numerical simulation methods of computational fluid dynamics. In the research, several basic problems of ball motion are analyzed by using computer graphic animation, experimental simulation with PIV, which is one of the most effective methods for measuring the complicated unsteady flow and numerical simulation methods of fluid flow.

2. Animation of the trajectory

It is possible to compute and animate the ball trajectory from a knowledge of fluid mechanics. Figure 1 shows an example of an animation frame produced by Tsuji in the current work. To make this animation, it is necessary first to solve the equations of motion of the ball, the ball bouncing on the table surface, and contact with the racket. The bouncing effects were solved using spring and dash-pot models in solid mechanics. The equation of the ball motion was solved in three-dimensional space, and AVS software was used to calculate the results as computer-graphics animations[2]. This software was developed by the Advanced Visual Systems Inc. of USA, and can produce three-dimensional animation in rectilinear or irregular coordinates. When the ball moves in air, it is subjected to many kinds of external force such as drag, lift, gravity, historical force, and so on. Historical force always varies with time. It is difficult to calculate the historical force of a sudden moving ball because the related air flow is three dimensional at high Reynolds number. The historical force is comparatively small in general and not enough to be formulated theoretically and experimentally. For simplicity, three main forces, i.e., drag, lift and gravity, are considered here to calculate the three-dimensional trajectories. Therefore, the equation of ball motion as shown in Fig. 2 (a) and (b) is described as,

Inertia force = Gravity + Drag + Lift

or

$$m\frac{d^2\vec{x}}{dt^2}=\vec{F}_g+\vec{F}_D+\vec{F}_L\,, \tag{1}$$

where the gravity force is

$$\vec{F}_g=-mg\vec{i}, \tag{2}$$

drag is

$$\vec{F}_D=-\frac{1}{8}C_D\rho\pi d^2|\vec{w}|\vec{w}, \tag{3}$$

and lift is

$$\vec{F}_L=\frac{1}{8}C_L\rho\pi d^2|\vec{w}|^2\frac{\vec{k}\times\vec{w}}{|\vec{k}\times\vec{w}|}\,. \tag{4}$$

In these equations, m is the mass of ball, $\vec{x}$ position vector, t time, $\vec{F}_g$ gravity force, $\vec{F}_D$ drag, $\vec{F}_L$ lift, g acceleration due to gravity, $\vec{i}$ unit vector in upward direction, C_D the drag coefficient, C_L the lift coefficient, ρ density of air, d diameter of ball, $\vec{w}$ relative velocity of ball to air, $\vec{k}$ unit vector normal to translation direction. The value of C_D is given by equation (5) and the values of constants C_0, C_1 and C_2 are listed in Table 1 according to Morsi & Alexander[3]:

$$C_D=C_0+\frac{C_1}{Re}+\frac{C_2}{Re^2}\,. \tag{5}$$

The equation of motion is the balance between the external forces and the inertia force. In the equation, ball size and mass and two kinds of velocity (translation and spin) are considered to be the important factors. The air density and temperature also play significant roles. For example, the players often feel that the ball trajectory is different in different air conditions even if they hit the ball in the same way. This is because differences of altitude or air temperature at different tournaments do affect the air flow around the ball. This kind of experience can be explained scientifically.

3. Collision (Bounce on the table and contact with the racket)

When a ball contacts a racket or a table surface, the dynamics of the rebound is necessary to analyze the trajectory. The velocity and direction of the ball after bouncing are estimated from known data such as the incident velocities (translation and spin), the incident angle, and the coefficients of restitution and friction between ball and bat or table. Figure 3 shows video images of a ball bouncing on a bat with inverted rubber. This was recorded by Yamamoto's group with a super-high-speed video system at nine thousand frames per second. From such video images, we can clearly see the process of ball deformation and the change of ball rotation. It should be possible to say that the bouncing problem of a table tennis ball can be solved by considering the major deformation of the ball and the viscoelasticity of the rubber as it changes with the time because of the use of adhesive. But the bouncing problem is more complex in the actual situation, and it has not been completely solved scientifically. The principal reason is that the experimental data on the various factors are lacking.

4. Experimental simulation of inside and outside flows

Figure 4 is an example of the experimental results of the distribution of velocities inside and outside a hollow circular cylinder; this is a two-dimensional simulation model of a table tennis ball that suddenly starts rotating in a uniform flow. In the Figure, dimensionless time t is defined based on real time t* as

$$t=\frac{2wt^*}{d}. \qquad (6)$$

Here the flow velocity is measured by PIV (Particle Imaging Velocimetry) which is one of the most powerful tools for measuring complex unsteady flow: the flow velocity is visualized by seeding very small solid particles into the fluid. The techniques of PIV have the following merits: instantaneous whole flow field measurement, contact free measurement, easy extraction and processing of physical information through velocity information. Details of PIV have been given by Adrian (1991)[4], Kobayashi & Yamamoto (1995)[5], etc. Figure 4 shows the time evolution of fluid flow after an impulsive spin of the initially static cylinder in a uniform flow. The unsteady velocity fields of the present flows can be obtained from the PIV measuremental method. Although this is just an experimental simulation of the two-dimensional model, the mechanism of the flow behavior due to an abrupt change of the spin can be understood and a reasonable estimation of the actual three dimensional motions of the ball can be made. Techniques of three dimensional PIV are being developed now. With these techniques, it should be possible to measure the three dimensional velocity field, and to observe and analyze the interaction between the ball and the air flow. This investigation will be helpful to solve the problem of the player's perception of weight when he/she hits the ball strongly with high speed and spin.

5. Fluid force and numerical analysis

Numerical analysis is an effective tool to study the details of the inside flow and the outside flow. The governing equations of fluid flow such as the Navier-Stokes equation may be solved numerically by using a high speed computer. The inside flow is generated by the ball rotation and is a three-dimensional phenomenon because of the complexity of translation and rotation. The outside flow is controlled by interaction of the environmental air flow and ball motion. A two-dimensional case (hollow circular cylinder with sudden rotation in a uniform flow) was investigated numerically by Yamamoto's group, and some numerical results are given in the paper.

Figure 5 describes a time variation of velocity distributions of the inside flow, where u is velocity of the uniform flow, ω the rotation speed and r1 the inside radius. Calculated velocity distributions agree with PIV measurement results except in the regions near the center and near the wall.

Figure 6 shows the fluid forces acting on the ball. The wall shear stress of the inside flow plays an important role in the initial period, but decreases quickly [in Fig. 6 (a)]. The outside flow takes a longer time to reach a steady state in Fig. 6 (b) and (c), where the drag coefficient C_d and the lift coefficient C_L are defined according to Chew, etc.[6]. Figure 6 (b) shows the total drag C_d, which consists of the pressure drag C_{dp} and the viscous drag C_{ds}, first increases from 1.71 due to the sudden rotation and then decreases to 1.58, which is the steady solution of the flow. Spin also produces lift according to Figure 6(c), where C_L is the total lift coefficient, C_{Ld} is the lift coefficient due to pressure, C_{Ls} the lift coefficient due to viscous shear stress. It can be seen how complexly the fluid forces act on the cylinder moving relative to air.

Figure 7 gives flow patterns for the two-dimensional case and shows that rotation makes the wake boundary layer thinner. Vortices exist in the wake region.

It can be expected that the entire three-dimensional flow will be solved numerically, and the time variation of drag, lift and flow structure will be obtained in detail by means of super-computers in the near future. It will also be possible to give numerical methods for controlling the ball's motion.

6. Relation between mass and velocity

Figure 8 indicates the simulated results of the change in ball velocity due to the bounce on the table, as calculated by Tsuji. The motion process of the ball in air is calculated by equation (1) on the assumption of torque decay due to air viscosity and the contact process on the table is simulated with spring and dash-pot models. Figure 8(a) shows the variables used to describe the motion. From Figure 8(b), showing the velocity of ball before it is returned from 1 meter behind the table, there is an important relation between its mass and the velocity. In Figure 8 the ball is struck horizontally at 33.3 m/s (120km/h) (initial velocity) and given 100 rpm of initial spin, and it then bounces on the opponent court. From this figure it can be estimated that if the mass of regular ball (2.5g) be made lighter by 5% (0.13g), the final velocity V_3 becomes 25.0m/s (90.0km/h) slower, -1.6% less than the 25.4m/s (91.4km/h) of the regular ball. Since top-level players can probably recognize the difference of 0.1% in velocity, a reduction of 1.6% in velocity would give them a significant change, which would make the ball easier to return, and the rally would be longer.

7. Ball deformation and internal air pressure

The investigation of ball deformation and its relation with air pressure was carried out by Yamamoto. The internal pressure was computed based on the analysis of adiabatic change of the air inside the ball because the contact time of the bounce is very short. As shown in Figure 9(a), a ball with radius R is assumed to become depressed by Δr on the contact side; we can then estimate the volume change of the air inside the ball. The internal air pressure before and after bouncing are P_0 and P' respectively. Since the diameter of the regular ball is 38mm (R=19mm), if Δr=2mm during the bounce, the pressure increase becomes $P'/P_0=1.01$ ($\Delta r/R \approx 0.1$) as shown in Figure 9(b). Since this effect is related to the fatigue strength of the ball material, the manufacturers cannot ignore it, although it is not important for the player returning the ball.

8. Other basic problems

Many other important factors are involved in ball motion. For example, the effects of air viscosity, Reynolds number (defined by translation velocity and the spin ratio between spinning circumferential velocity and translation velocity), the fluctuations of the drag and lift forces due to production of the periodically turbulent vortex behind the ball, the irregular bounce due to non-sphericality of the ball and the roughness of table surface, and virtual mass effect by the air flows inside and outside the ball. Therefore, it is rather difficult to solve the problem of actual ball motion completely owing to lack of experimental and theoretical information.

9. Conclusion

If we simulate the complex trajectory of a table tennis ball theoretically, and visualize the results by an animation as in Figure 1, the scientific basis for the mechanism of fluid motion will be helpful to explain, for example, the relation between human physical ability and the limitations on returning ball, the results of research of ball-striking skill at impact, foot work and body work, the method of muscular training, the enjoyment of both players and spectators, and also the improvement or development of the rules of the sport.

The study of complex forces of the air flow acting on the ball is very important for simulating the ball trajectory. It is possible to obtain these unsteady fluid forces by solving governing equations of the air flow numerically with super-computers.

As we know, we can observe animation results very easily only by inputting suitable values of several variables into the computer, then we can investigate the effect of each variable and the correlations among them. The computer animation is excellent as a research tool; we therefore hope that this kind of research with computer simulation, PIV measurement and graphical animation will contribute not only to solving the problems of table tennis science but also to those of other sports.

Acknowledgement

The authors would like to express their great thanks to Professor Kei-ichi Sato, Kanazawa Institute of Technology, and Professor Takeharu Etoh, Kinki University for their kind cooperation in the use of high speed video camera systems.

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Table 1 Coefficients for C_D in equation (1)

(From Morsi & Alexander)

Re	C0	C1	C2
Re<0.1	0	24	0
0.1<Re<1.0	3.69	22.73	0.0903
1.0<Re<10	1.222	29.1667	-3.889
10<Re<102	0.6167	46.5	-116.67
102<Re<103	0.3644	98.33	-2778.0
103<Re<5×103	0.3571	148.62	-47500.0

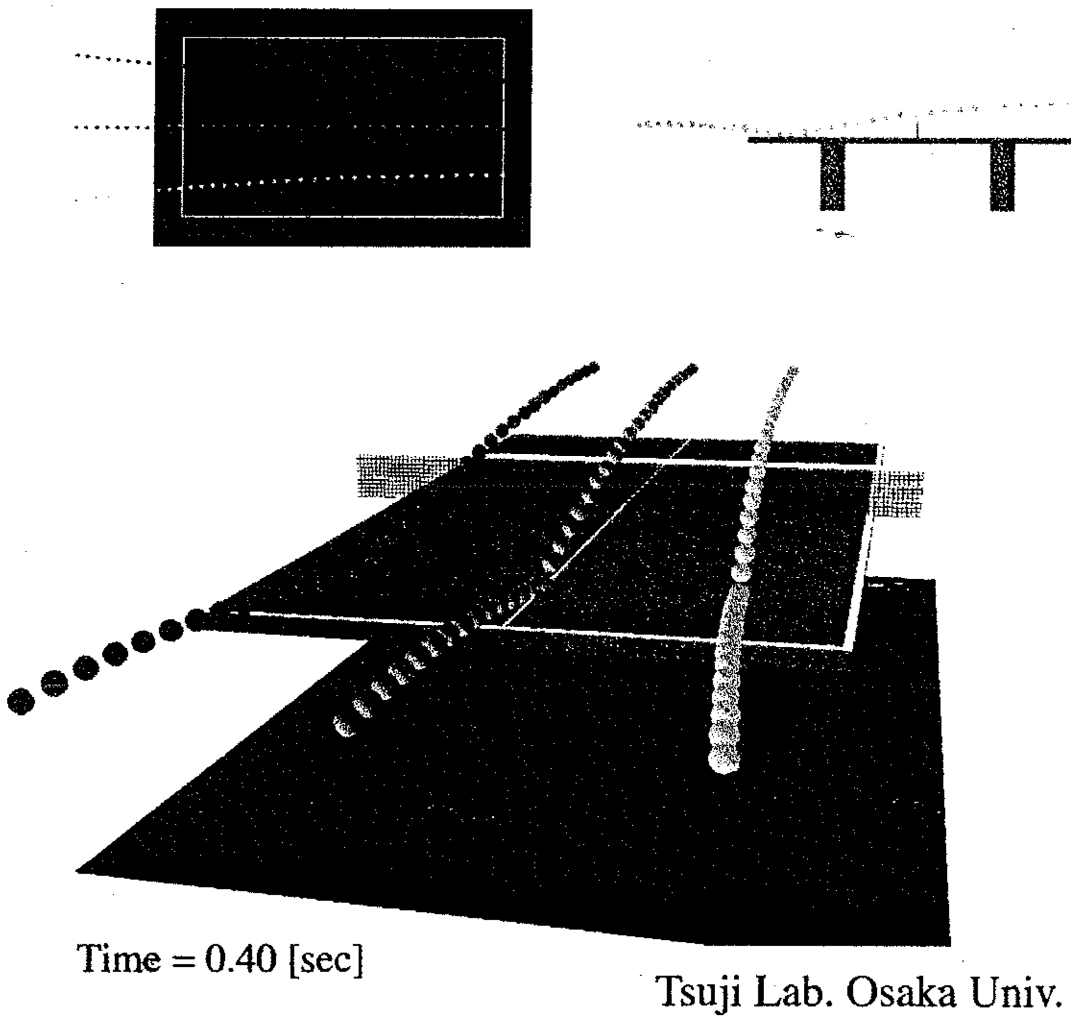
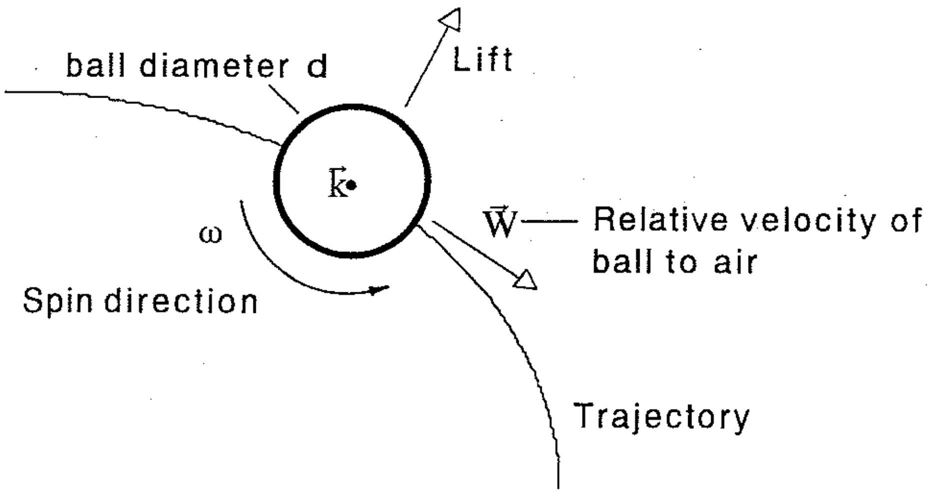
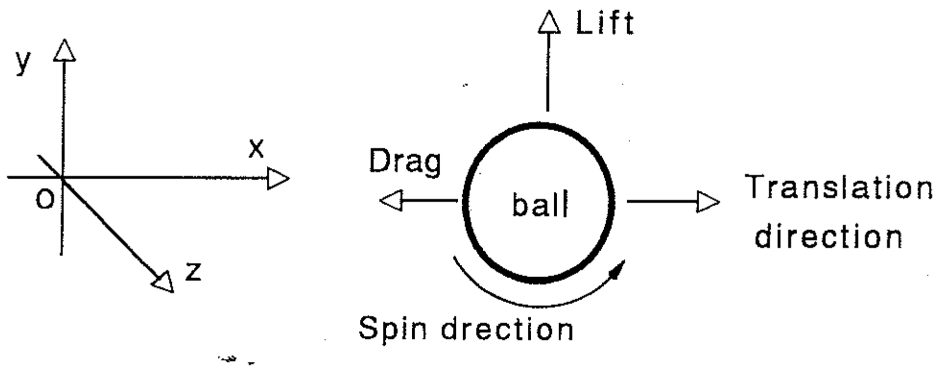


Figure 1. Computer animation of ball trajectory

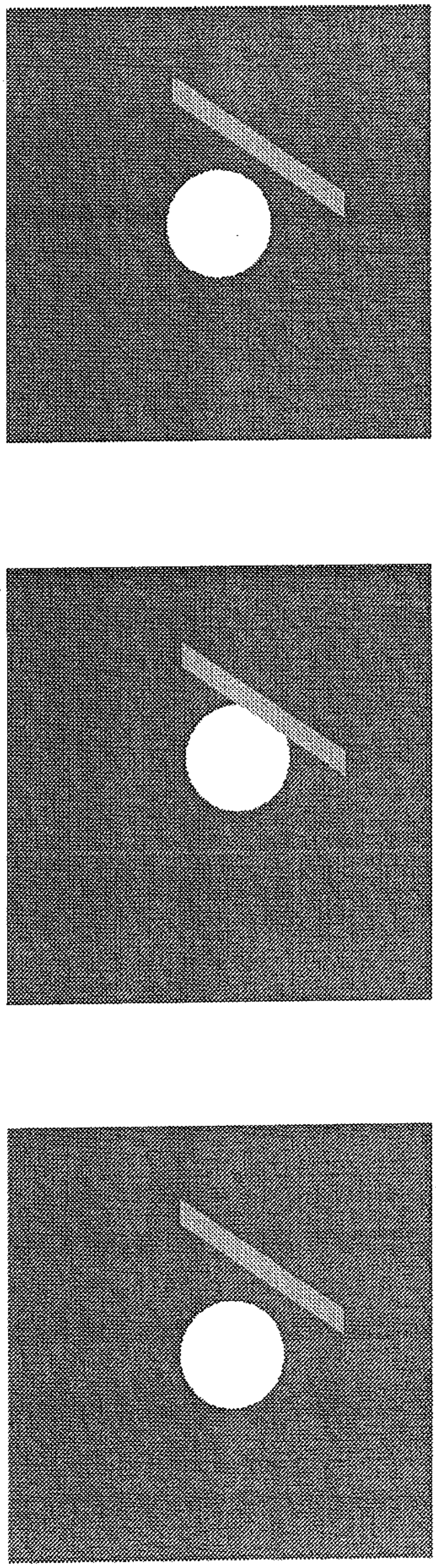


(a) Relative velocity changes with trajectory



(b) Coordinate system

Figure 2. Coordinate system of ball motion



(a) 9/9000(s) before hit

(b) on contact

(c) 6/9000(s) after hit

Figure 3. Images by super high speed video when a bat hits a drive ball

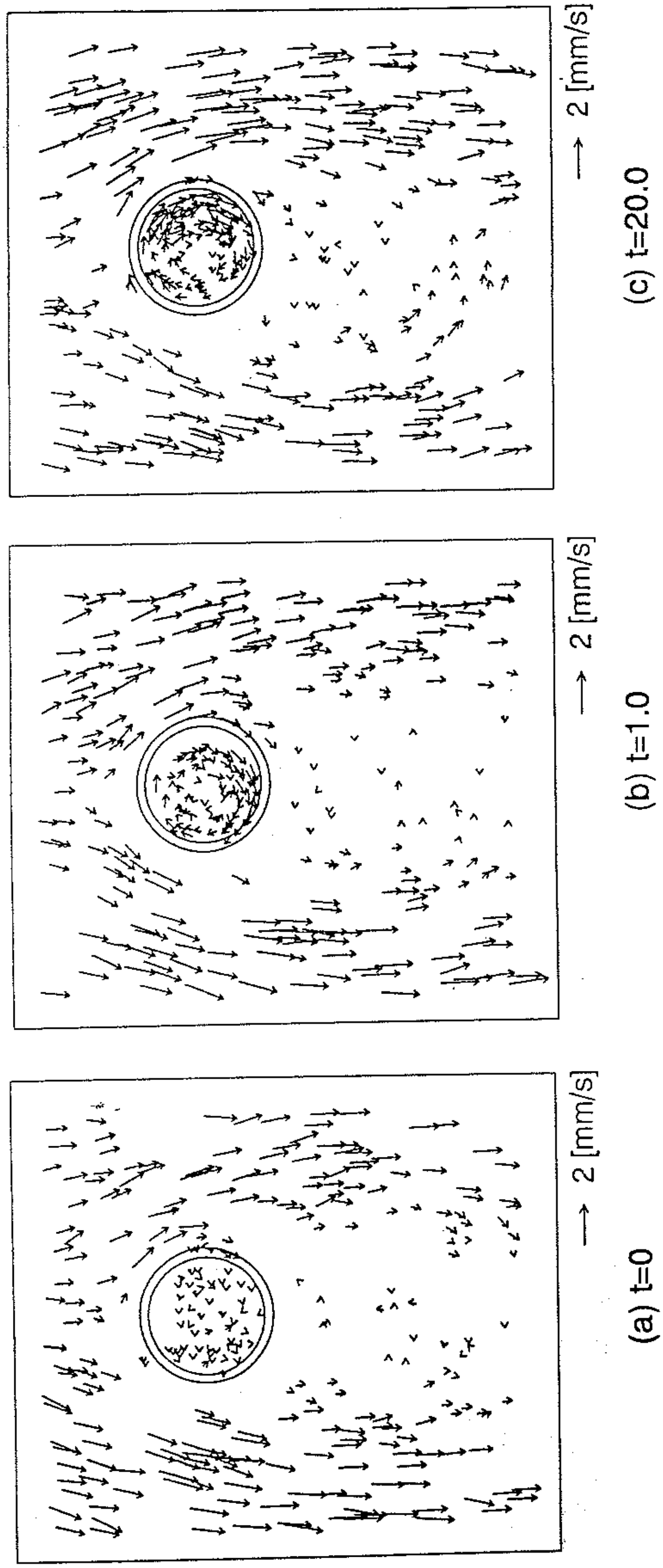


Figure 4. Velocity vectors from PIV

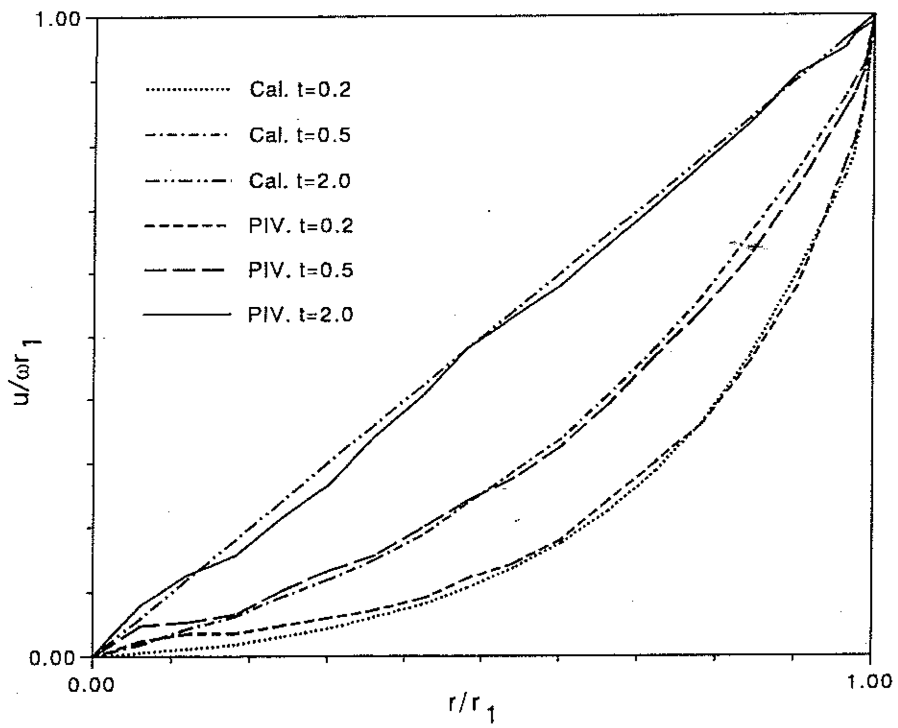
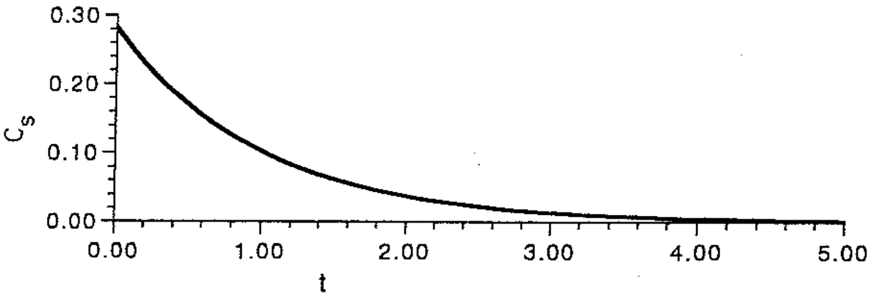
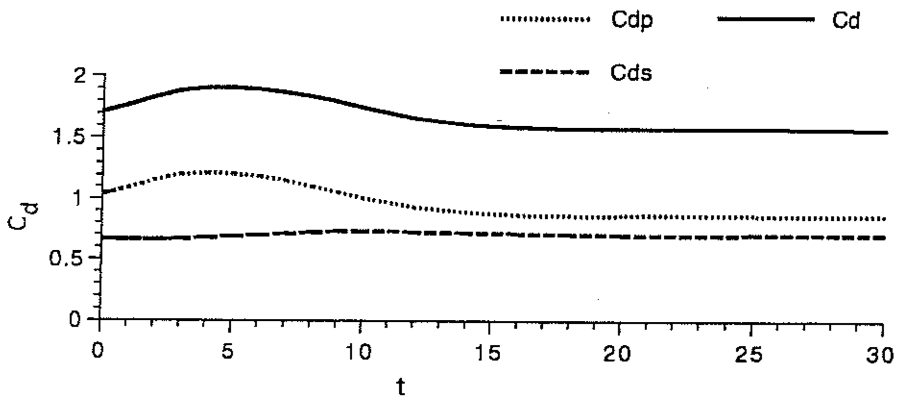


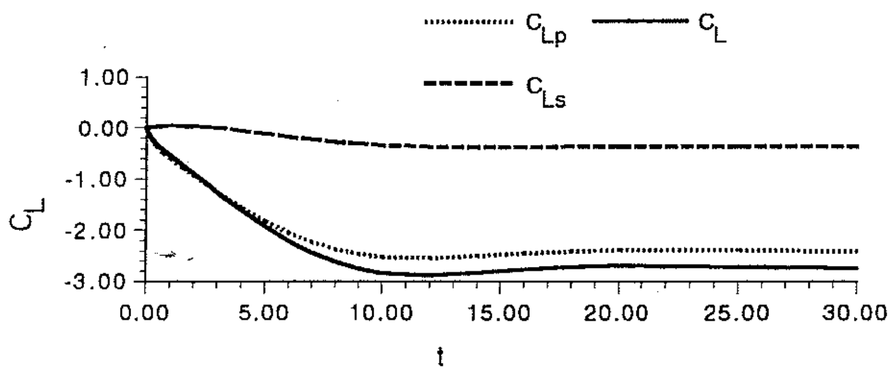
Figure 5. Time variation of velocity distribution of the inside flow from calculation and PIV measurement



(a) Time variation of wall shear stress of the inside flow

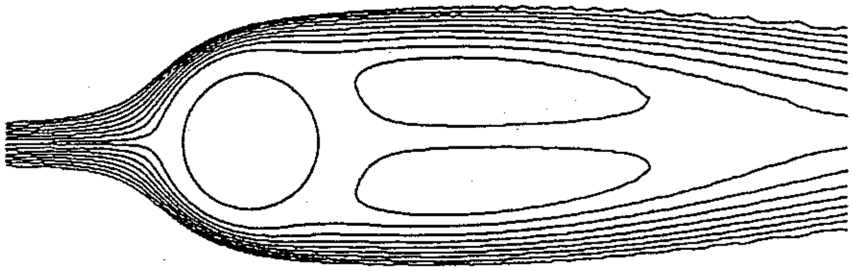


(b) Time variation of drag

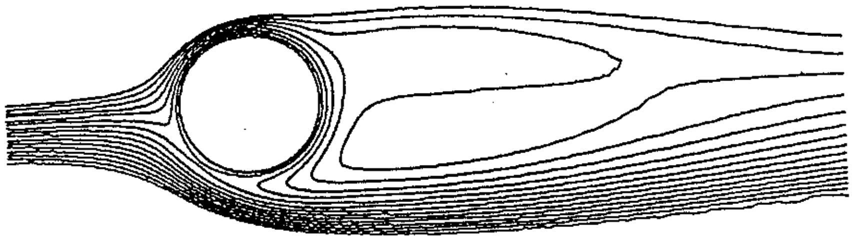


(c) Time variation of lift

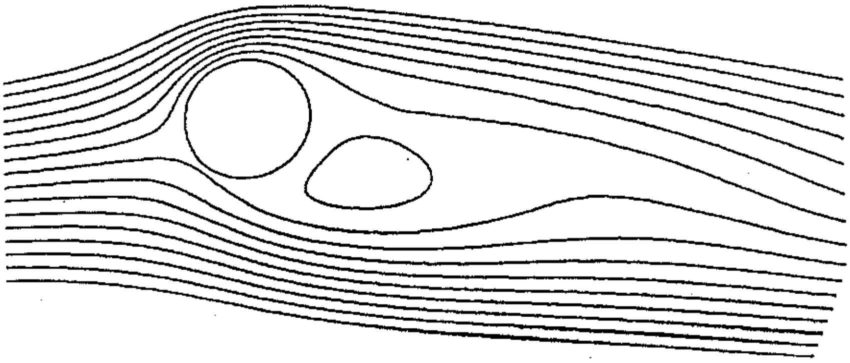
Figure 6. Time variation of fluid forces



(a) t=0

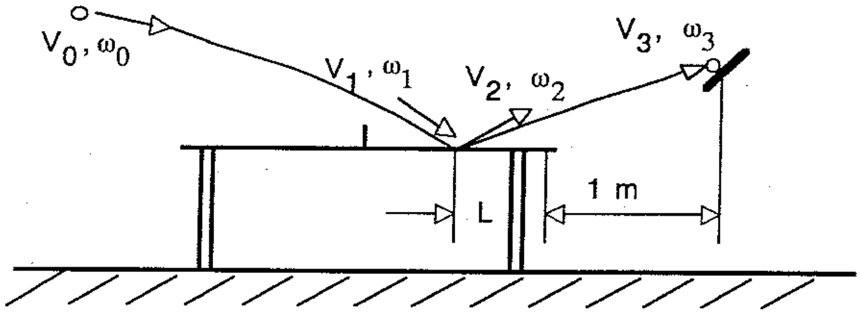


(b) t=1.0

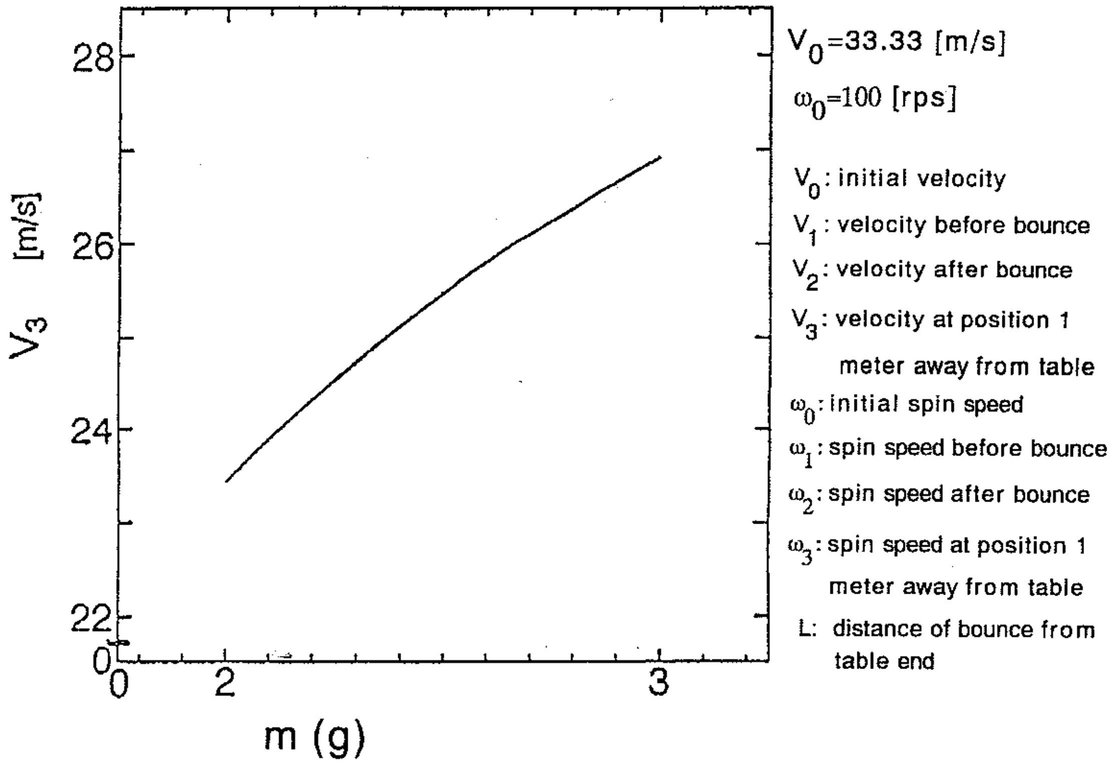


(c) t=20.0

Figure 7. Streamlines of the outside flows

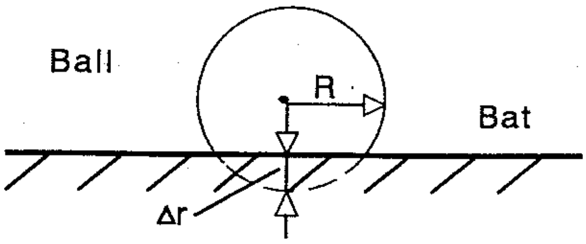


(a) Explanation of symbols

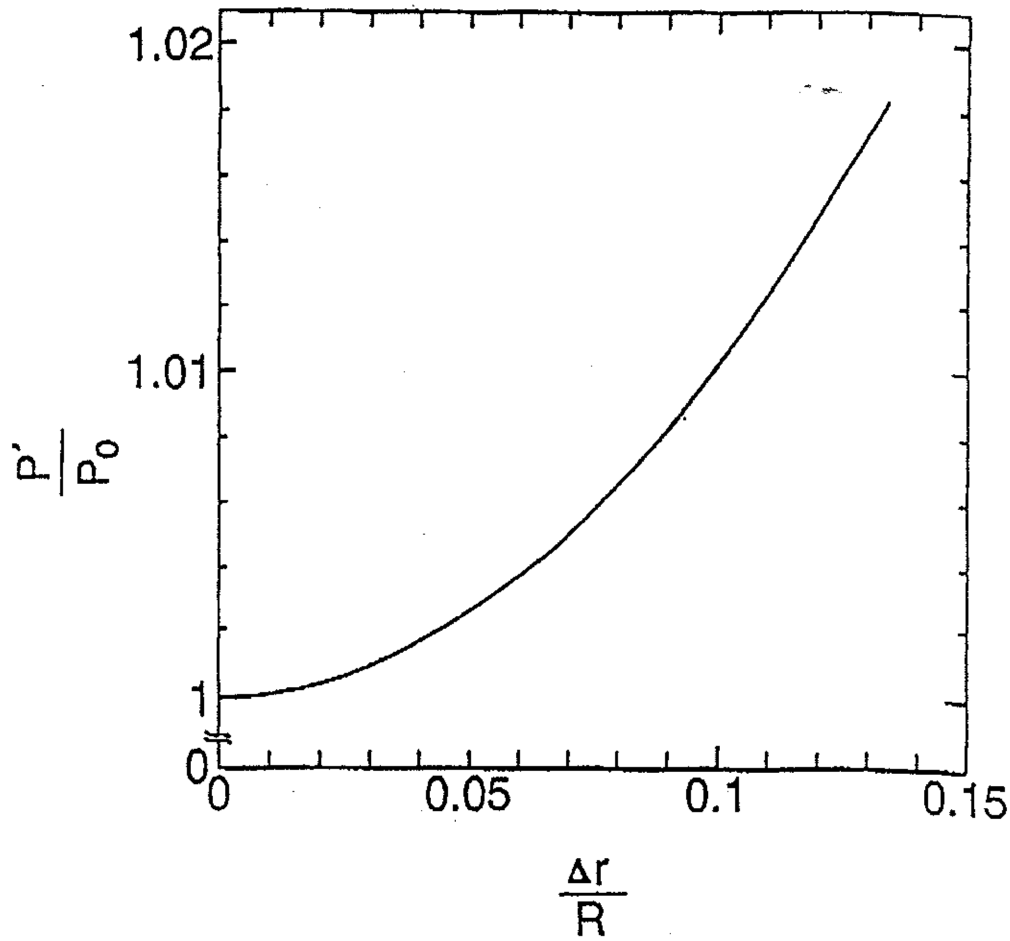


(b) Relation between m and V_3

Figure 8. Relation of ball mass m and speed V_3



(a) Explanation of symbols



(b) Relation between Δr/R and P'/P_0

Figure 9. Relation of ball deformation Δr/R and air pressure ration P'/P_0