Elaborating a rule for game handicap in table tennis: a probabilistic approach

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Abstract: in sporting event, the installation of a game handicap rule is an often delicate problem. Some sports as tennis or golf solve the question by defining a nomenclature of classification based on the notion of handicap. Therefore, numerous table tennis clubs organize tournaments based on various game handicap formulas. We offer a probabilistic approach allowing defining a rational, fair and simple rule of game handicap.

Keywords: games with null sum, probability of win, game handicap.

1. INTRODUCTION

The development of a handicap rule is a compulsory step in every sport. Indeed, levelling the playfield by rebalancing chances of victory for two players is a necessity during training as well as in some competitions. Regarding some sports such as tennis [1] or golf, this notion of handicap is where rankings come from. In table tennis, the current ranking method (a number of points), like the previous one, aren't built on a notion of handicap. As far as a ranking procedure exists, a probabilistic distance between two ranked players can be deduced and this inequality in victory chances can be compensated by a handicap rule. We propose a method aiming at estimating probabilistic distances between ranks in table tennis. These distances allow the development of a handicap rule.

2. PROBABILITIES TO WIN THE MATCH

In table tennis, each match during an official competition is taken into account to establish the ranking. Depending on who wins the match and the difference between the two players' rankings, the winner's number of points increases, while the loser's decreases according to a table defined by the French Federation of Table Tennis (FFTT).

Table 1 (source FFTT-2013)

Ranking Gap	The Best Wins	The Worst Loses	The Worst Wins	The Best Loses
0-24	6	-5	6	-5
25-49	5,5	-4,5	7	-6
50-99	5	-4	8	-7
100-149	4	-3	10	-8
150-199	3	-2	13	-10
200-299	2	-1	17	-12,5
300-399	1	-0,5	22	-16
400-499	0,5	0	28	-20
500+	0	0	40	-29

In game theory [2], we consider that a game is balanced if the chances of gain are null for each of the two participants (« zero-sum game »). More exactly, when two players A and B compete and the probability of A winning is noted P_A (with $P_A \geq 1/2$), then the match is considered a zero-sum game if :

$$P_A K_A + (1 - P_A) K_B = 0$$

Then $$P_A = K_B / \left(K_A - K_B\right)$$

From this principle, we are able to work out for any K_A (number of points won by A if A wins the match) and K_B (number of points lost by A if B wins the match) of table 1 and thus for any ranking difference between two players, the probability for the better ranked player to win (P_A).

Usually (in a zero-sum game), for the same ranking discrepancies, the chances of normal victory (the best player wins) and abnormal defeat (the worst player wins) should sum up to 1. The same should be for the probabilities of abnormal victories and normal defeats. This isn't exactly the case. Particularly for a points difference between 0 and 24, we should have probabilities all equal to ½. The current points awarding isn't exactly the solution to a zero-sum game.

Nevertheless, these calculus give a frame to every possibility. It is then only natural to take as probability the middle of each interval as shown in Fig. 1.

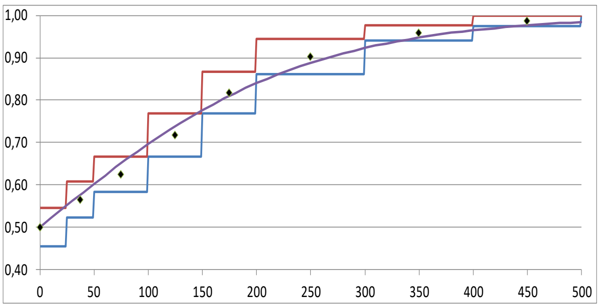

Fig. 1 Probabilities of average victories smoothened by Bradley-Terry model [3]

3. PROBABILITY TO SCORE A POINT

In table tennis, a traditional match is played best of five games of 11 points each. We can connect the probability of winning a match with handicap $P(p,H)$ according to the probability, p, of scoring a point [4]:

$$P(p,H) = P_{GM}(p,H)^3 + 3P_{GM}(p,H)^3\left(1-P_{GM}(p,H)\right) + 6P_{GM}(p,H)^3\left(1-P_{GM}(p,H)\right)^2$$

with $P_{GM}(p,H)$, the probability to win a game with a handicap H :

$$P_{GM}(p,H) = \sum_{i=0}^{9-H} C_{10+i}^{i} p^{11}(1-p)^i + C_{20-H}^{10-H}\sum_{i=10}^{\infty} 2^{i-10} p^{i+2}(1-p)^{i-H}$$

H = 1, 2, etc. this means that the least ranked player starts each game 1 or 2 points… ahead depending on his probability (a priori) to win the game (without handicap).

4. ELABORATION OF GAME HANDICAP STEP BY STEP

(1) We estimate $P(p,0)$ for each ranking difference as defined by the FFTT point awarding table (made with Bradley-Terry model). (2) Solving the equation $P(p,H)$ when H=0, we estimate the probability, p, to score a point depending on the probability to win the game without handicap $P(p,0)$. (3) Solving the equation $P(p,H)$ when H=1, 2,…, we estimate the probability to score a point, p_H, with a handicap H leading to a probability of winning the game $P(p_H,H) = ½$. (4) When H=0, we estimate the probability to win the game without handicap $P(p_H,0)$. (5) Comparing $P(p,0)$ and $P(p_H,0)$, we get the ranking points differences for each handicap H.

5. RESULTS

Fig. 2 shows that the relation between ranking difference and the “right” handicap is linear. We can then propose a very simple game handicap.

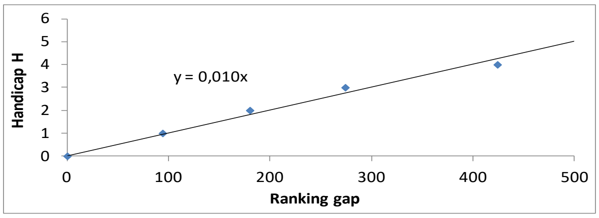

Fig. 2 Relation between ranking difference and positive game handicap

Table 2 Theoretical rule of positive handicap

ranking gap	0-49	50-149	150-249	250-349	350-449	450-549	550-649	650 and more
H	0	+1	+2	+3	+4	+5	+6	+7

For instance, a 1300 playing against a 1430 would start each game 1 point ahead (+1). Handicap shouldn’t increase indefinitely. Indeed, a +11 handicap would always make the least ranked player win. We state that a +7 handicap is the maximum acceptable for players.

6. CONCLUSION

Implementing a game handicap principle must not be a subjective operation coming from a trial and error process. It is a direct consequence of the ranking procedure itself. A good handicap rule must “wipe the slate clean”.

The handicap rule proposed is based on a victory probability equal for both players, whatever their ranking difference. We may wonder what would be the interest for a better ranked player to participate in this kind of match. Indeed, this “slate cleaning” advantages the weakest player.

To keep an ascending position, more symbolic than really significant, we suggest letting a slight advantage to the best ranked player. This way, the “advantage” rule would be the following:

Table 3 Chosen handicap rule

ranking gap	0-99	100-199	200-299	300-399	400-499	500-599	600-699	700 and more
H	0	+1	+2	+3	+4	+5	+6	+7

For instance, if the ranking difference between two players is 215 points, the least ranked player will start 2 points ahead (+2).

NB : this handicap rule usually leads to shorter matches than a match opposing two equally ranked players.

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